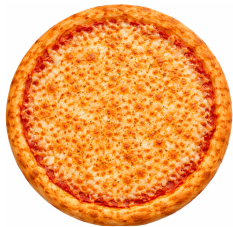
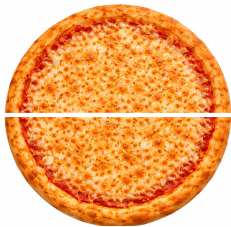


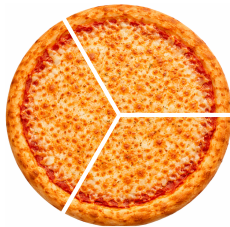
Cutting equal parts



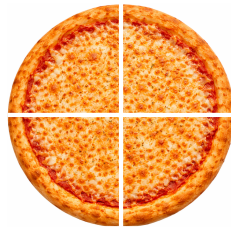
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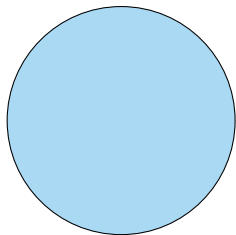
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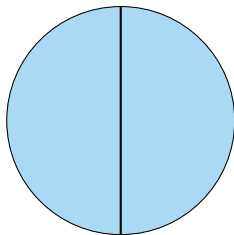
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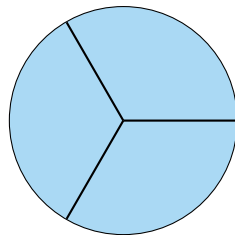
4



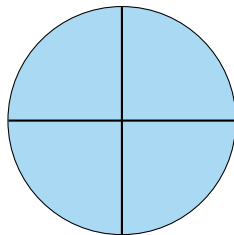
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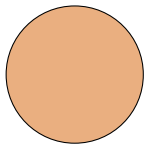
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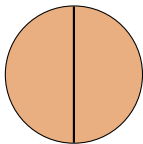
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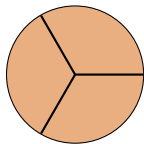
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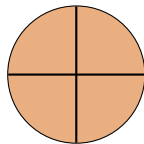
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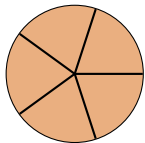
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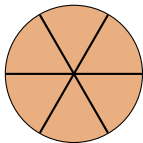
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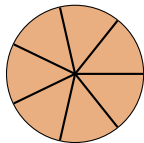
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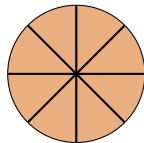
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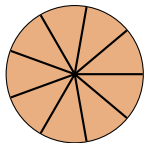
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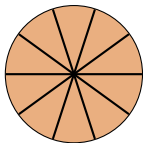
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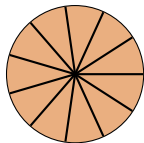
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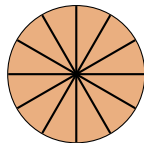
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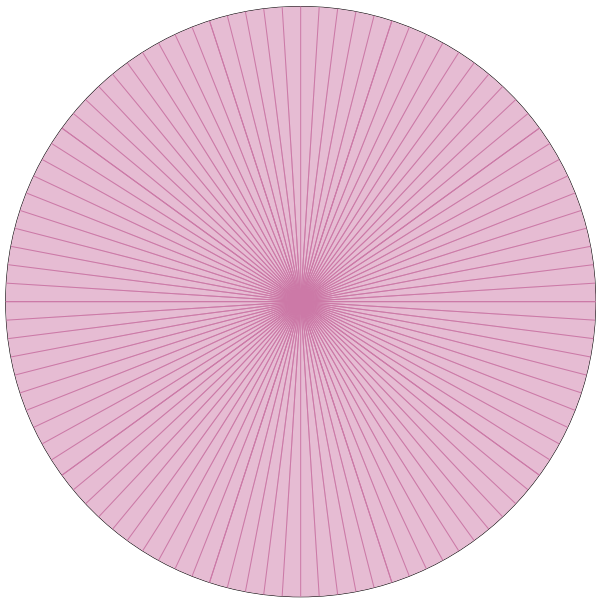
10



11



12



100



1



2



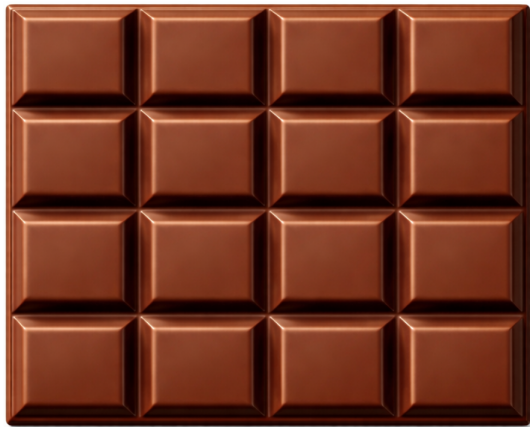
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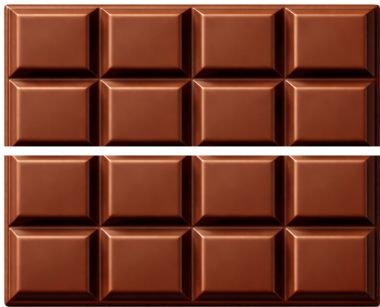


4



5

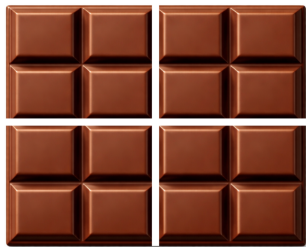




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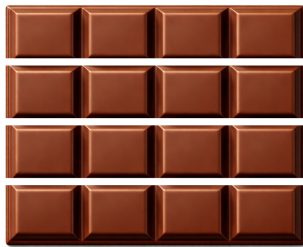
2



4



4



4



8

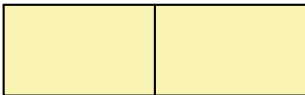


8





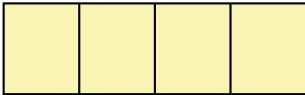
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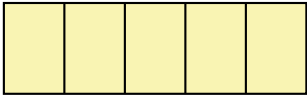
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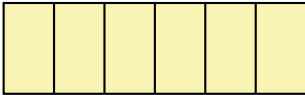
3



4



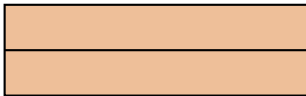
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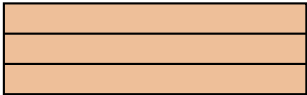
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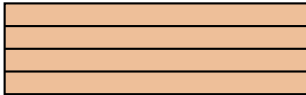
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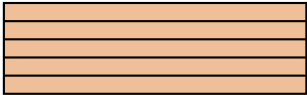
2



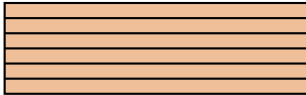
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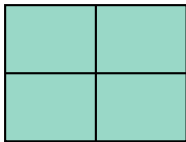
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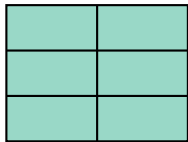
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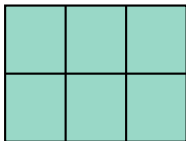
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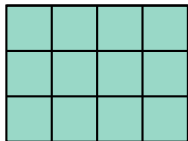
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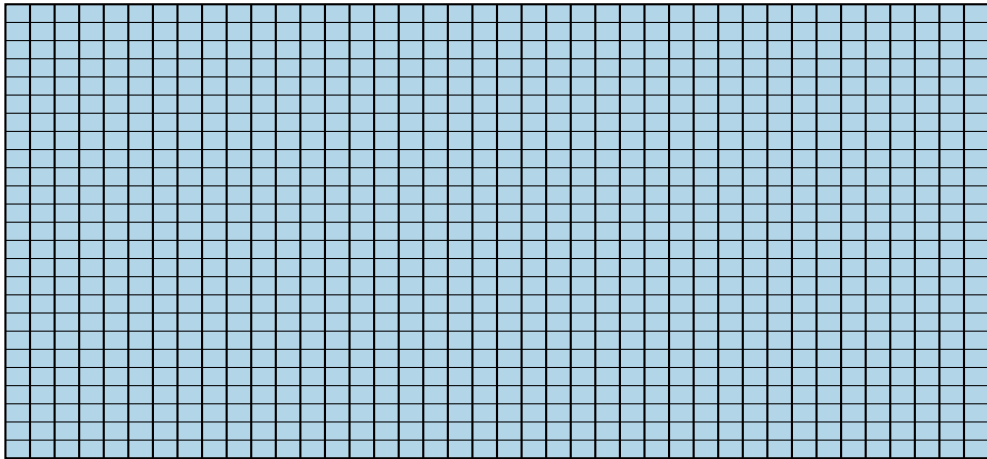
6



6

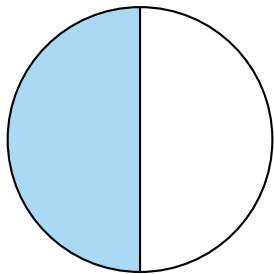


12

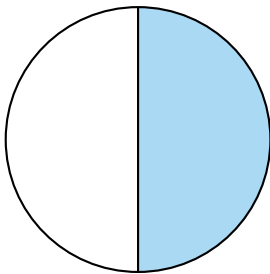


1000

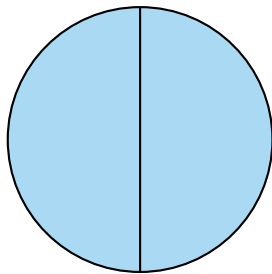
Taking some of the equal parts



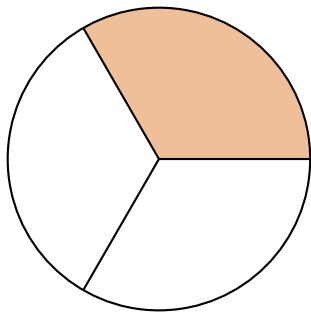
$$\frac{1}{2}$$



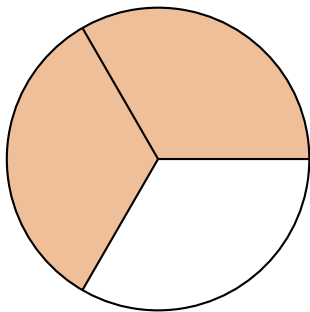
$$\frac{1}{2}$$



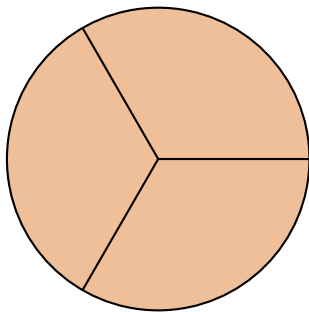
$$\frac{2}{2}$$



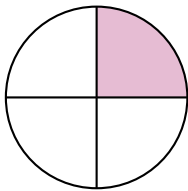
$$\frac{1}{3}$$



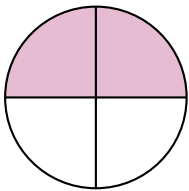
$$\frac{2}{3}$$



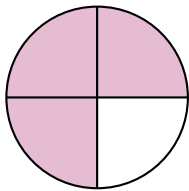
$$\frac{3}{3}$$



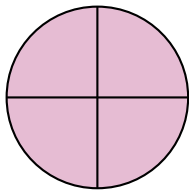
$$\frac{1}{4}$$



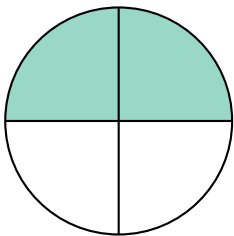
$$\frac{2}{4}$$



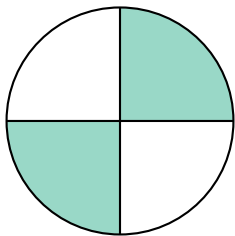
$$\frac{3}{4}$$



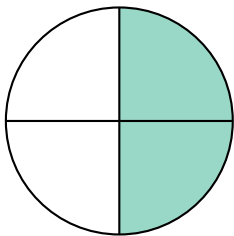
$$\frac{4}{4}$$



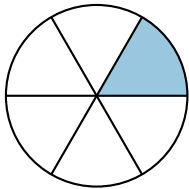
$$\frac{2}{4}$$



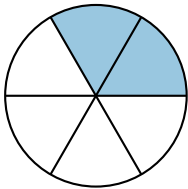
$$\frac{2}{4}$$



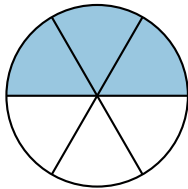
$$\frac{2}{4}$$



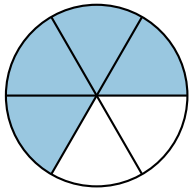
$\frac{1}{6}$



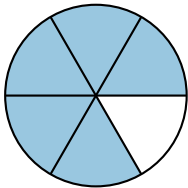
$\frac{2}{6}$



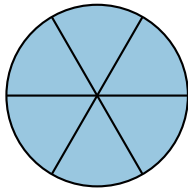
$\frac{3}{6}$



$\frac{4}{6}$

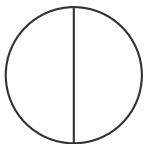


$\frac{5}{6}$

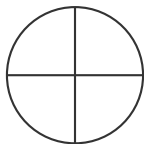


$\frac{6}{6}$

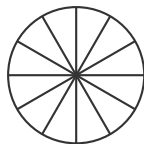
$$\frac{5}{9}$$



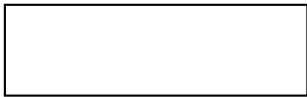
$$\frac{0}{2}$$



$$\frac{0}{4}$$



$$\frac{0}{12}$$



$$\frac{0}{1}$$



$$\frac{0}{6}$$

$\frac{4}{4}$



$\frac{3}{4}$

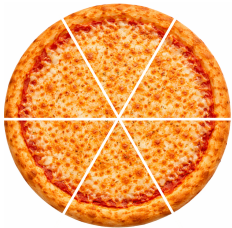


$\frac{2}{4}$

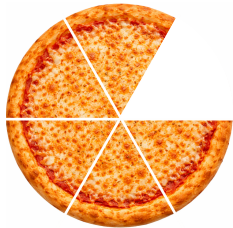


$\frac{1}{4}$

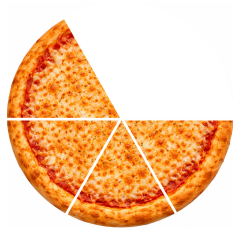




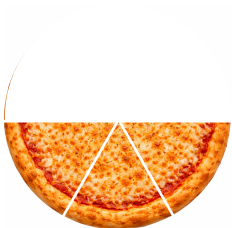
$$\frac{6}{6}$$



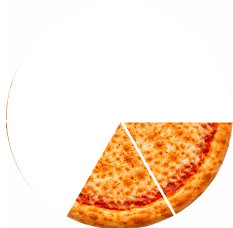
$$\frac{5}{6}$$



$$\frac{4}{6}$$



$$\frac{3}{6}$$

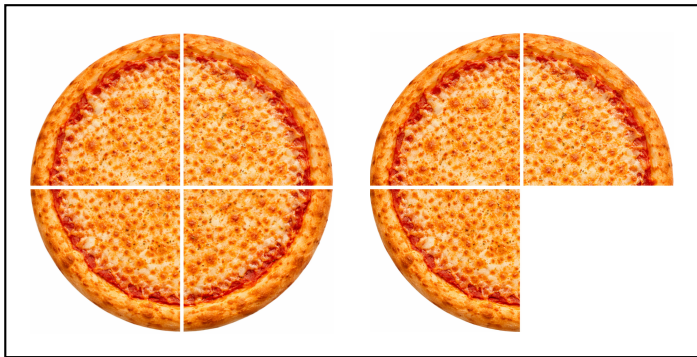


$$\frac{2}{6}$$

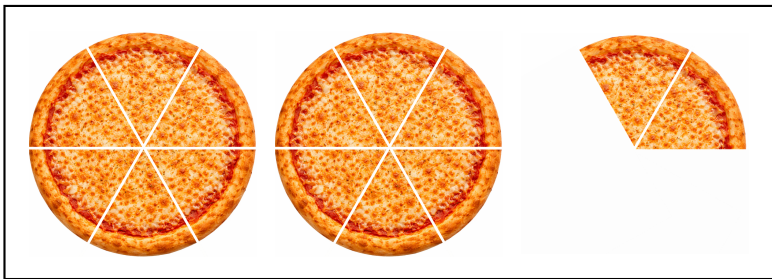


$$\frac{1}{6}$$

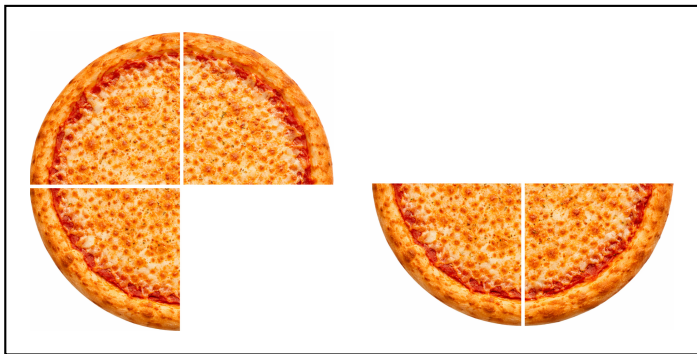
Many of the equal parts



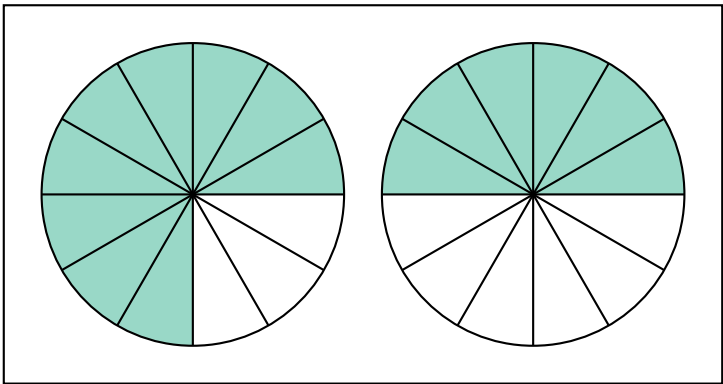
$$\frac{7}{4}$$



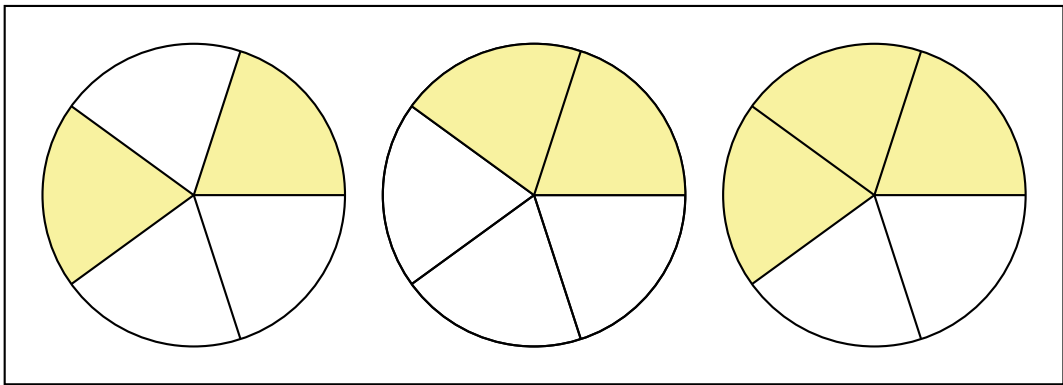
$$\frac{14}{6}$$



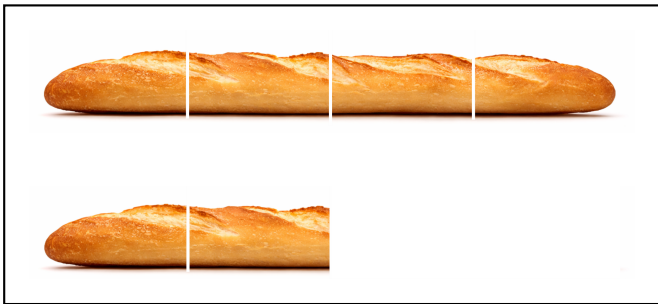
$$\frac{5}{4}$$



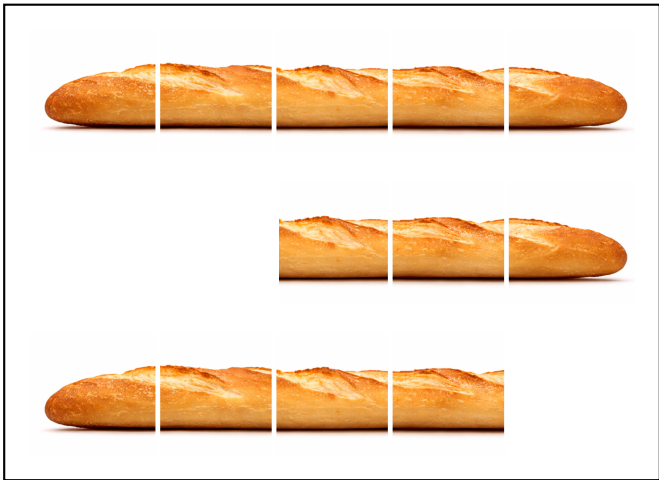
$$\frac{15}{12}$$



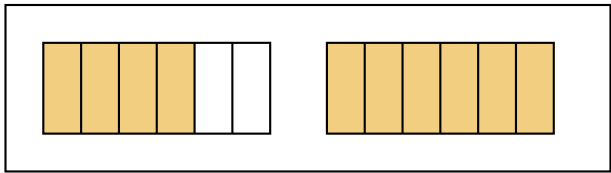
$$\frac{7}{5}$$



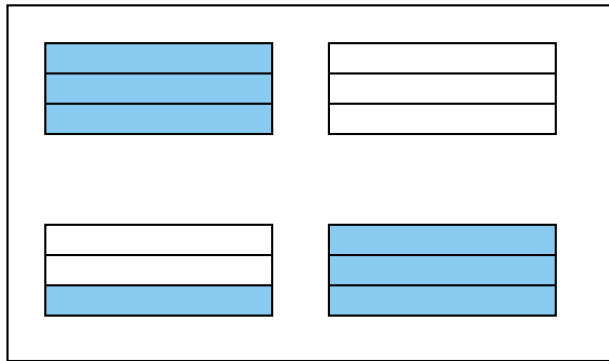
$$\frac{6}{4}$$



$$\frac{12}{5}$$

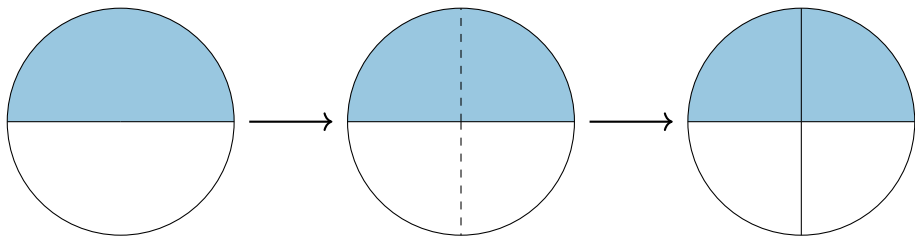


$$\frac{10}{6}$$



$$\frac{7}{3}$$

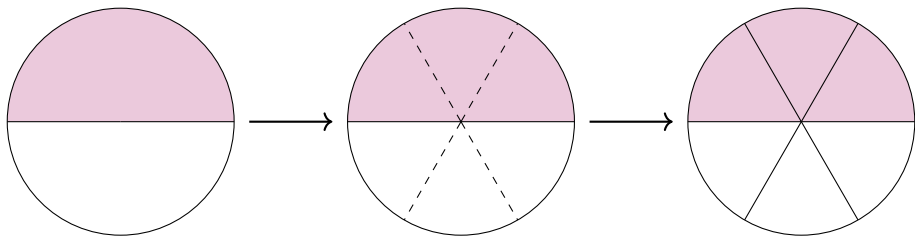
Cutting further



$$\frac{1}{2} = \frac{2}{4}$$

$\times 2$

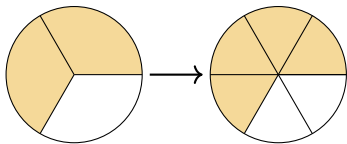
$\times 2$



$$\frac{1}{2} = \frac{3}{6}$$

$\times 3$

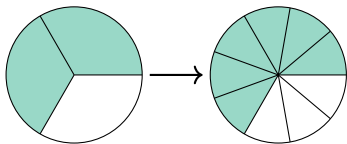
$\times 3$



$$\frac{2}{3} = \frac{4}{6}$$

Diagram illustrating the conversion of $\frac{2}{3}$ to $\frac{4}{6}$ using multiplication by 2:

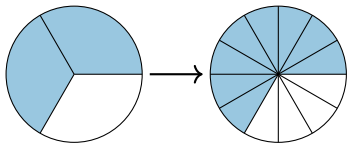
- Numerator: $2 \times 2 = 4$ (indicated by an orange arrow and $\times 2$ above the fraction bar)
- Denominator: $3 \times 2 = 6$ (indicated by an orange arrow and $\times 2$ below the fraction bar)



$$\frac{2}{3} = \frac{6}{9}$$

Diagram illustrating the conversion of $\frac{2}{3}$ to $\frac{6}{9}$ using multiplication by 3:

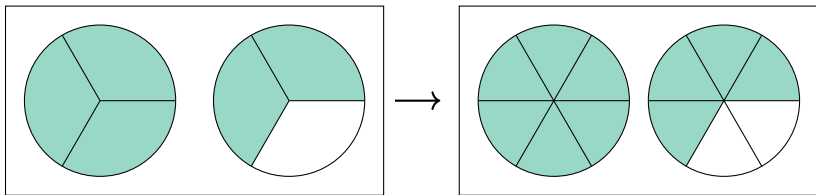
- Numerator: $2 \times 3 = 6$ (indicated by a teal arrow and $\times 3$ above the fraction bar)
- Denominator: $3 \times 3 = 9$ (indicated by a teal arrow and $\times 3$ below the fraction bar)



$$\frac{2}{3} = \frac{8}{12}$$

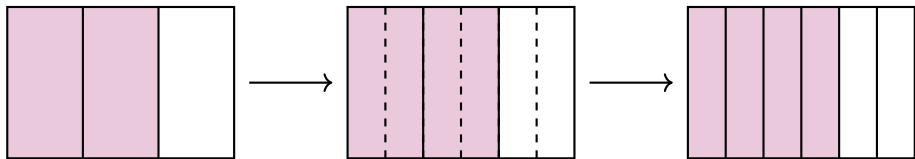
Diagram illustrating the conversion of $\frac{2}{3}$ to $\frac{8}{12}$ using multiplication by 4:

- Numerator: $2 \times 4 = 8$ (indicated by a blue arrow and $\times 4$ above the fraction bar)
- Denominator: $3 \times 4 = 12$ (indicated by a blue arrow and $\times 4$ below the fraction bar)



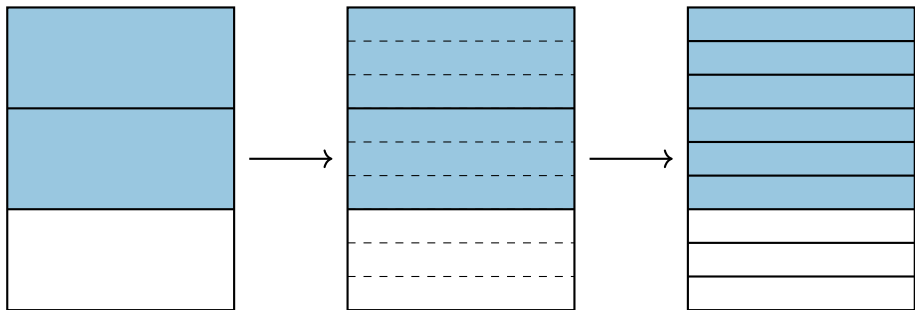
$$\frac{5}{3} = \frac{10}{6}$$

The equation is annotated with green arrows and text. A curved arrow points from the numerator 5 to the numerator 10, labeled $\times 2$. Another curved arrow points from the denominator 3 to the denominator 6, labeled $\times 2$.

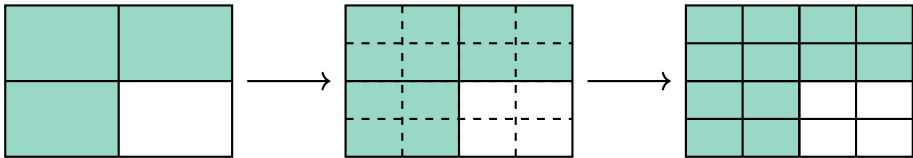


$$\frac{2}{3} = \frac{4}{6}$$

The equation is annotated with pink arrows and text to show the scaling process. A curved arrow points from the numerator 2 to the numerator 4, with the text $\times 2$ above it. Another curved arrow points from the denominator 3 to the denominator 6, with the text $\times 2$ below it.



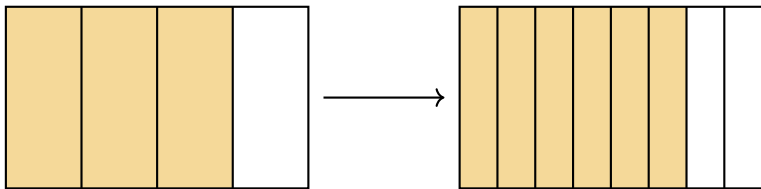
$$\begin{array}{ccc} & \times 3 & \\ 2 & \xrightarrow{\quad} & 6 \\ \frac{2}{3} & = & \frac{6}{9} \\ 3 & \xrightarrow{\quad} & 9 \\ & \times 3 & \end{array}$$



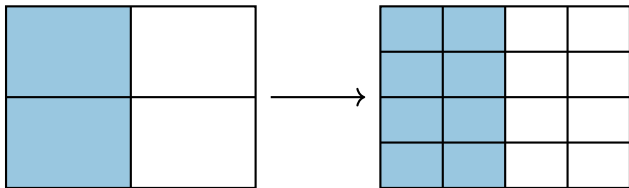
$$\frac{3}{4} = \frac{12}{16}$$

Diagram illustrating the conversion of the fraction $\frac{3}{4}$ to $\frac{12}{16}$ by multiplying both the numerator and denominator by 4:

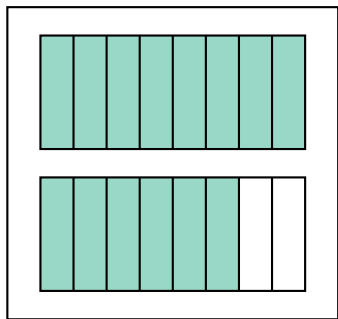
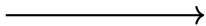
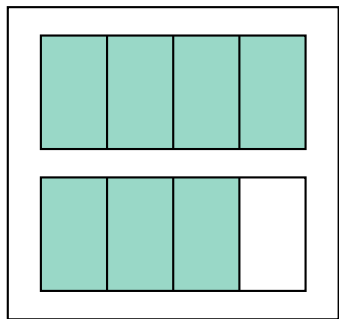
- Numerator: $3 \times 4 = 12$ (indicated by a curved arrow and $\times 4$)
- Denominator: $4 \times 4 = 16$ (indicated by a curved arrow and $\times 4$)



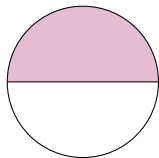
$$\begin{array}{ccc} & \times 2 & \\ 3 & \xrightarrow{\quad} & 6 \\ \frac{3}{4} & = & \frac{6}{8} \\ 4 & \xrightarrow{\quad} & 8 \\ & \times 2 & \end{array}$$



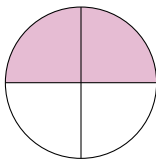
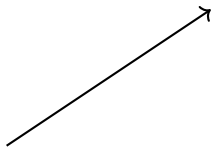
$$\begin{array}{ccc} & \times 4 & \\ 2 & \xrightarrow{\quad} & 8 \\ \hline 4 & = & 16 \\ & \xleftarrow{\quad} & \\ & \times 4 & \end{array}$$



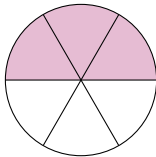
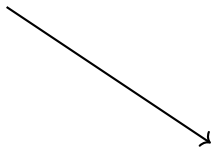
$$\frac{7}{4} \overset{\times 2}{=} \frac{14}{8}$$



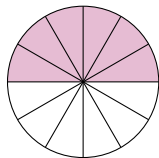
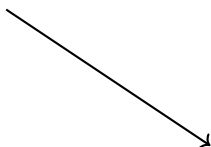
$$\frac{1}{2}$$



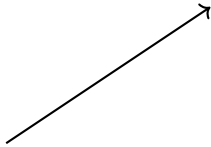
$$\frac{2}{4}$$

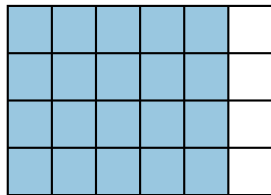
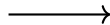
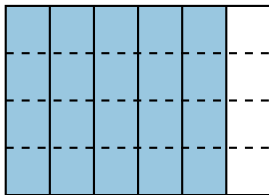
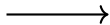
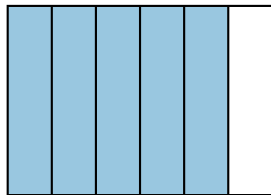


$$\frac{3}{6}$$



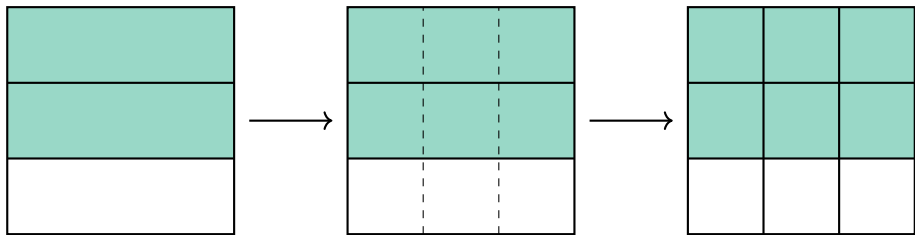
$$\frac{6}{12}$$





$$\frac{5}{6} = \frac{20}{24}$$

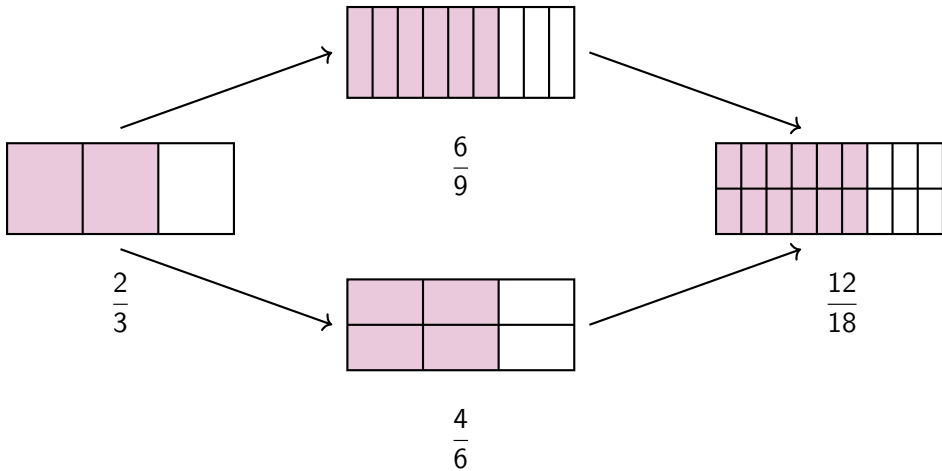
$\times 4$
 $\curvearrowright$
 $\times 4$



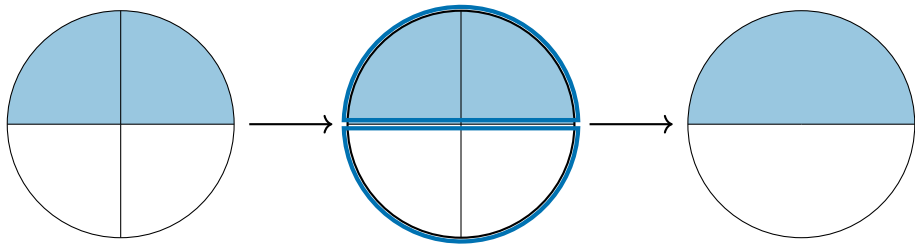
$$\frac{2}{3} = \frac{6}{9}$$

The diagram shows the multiplication of both the numerator and denominator by 3 to convert the fraction:

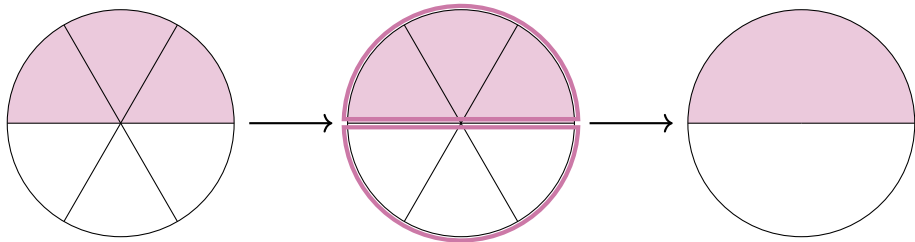
- Numerator: $2 \xrightarrow{\times 3} 6$
- Denominator: $3 \xrightarrow{\times 3} 9$



Bigger slices



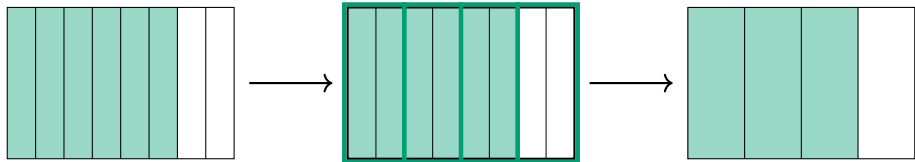
$$\begin{array}{ccc} & :2 & \\ 2 & \curvearrowright & 1 \\ \frac{2}{4} & = & \frac{1}{2} \\ & \curvearrowleft & \\ & :2 & \end{array}$$



$$\frac{3}{6} = \frac{1}{2}$$

Diagram illustrating the simplification of the fraction $\frac{3}{6}$ to $\frac{1}{2}$ using division by 3:

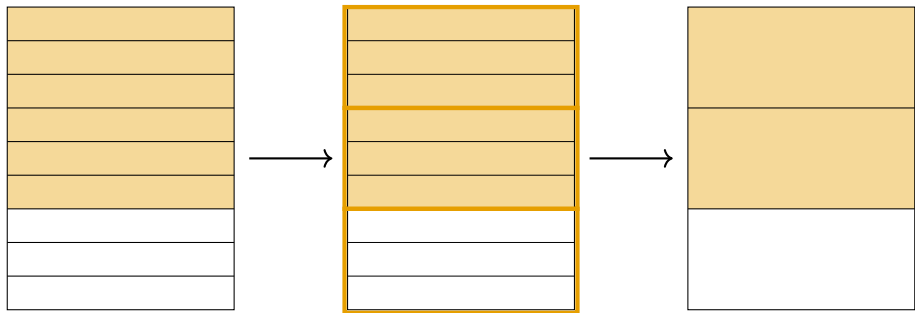
- The numerator 3 is divided by 3 to get 1.
- The denominator 6 is divided by 3 to get 2.



$$\frac{6}{8} = \frac{3}{4}$$

The diagram shows the simplification process with green annotations:

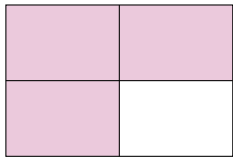
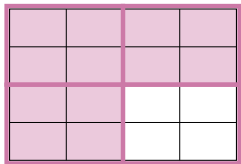
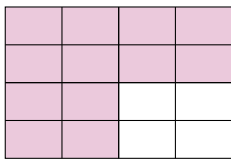
- A green arrow points from the numerator 6 to the numerator 3, with the label $:2$ above it.
- A green arrow points from the denominator 8 to the denominator 4, with the label $:2$ below it.



$$\frac{6}{9} = \frac{2}{3}$$

The diagram shows the simplification process with arrows and labels:

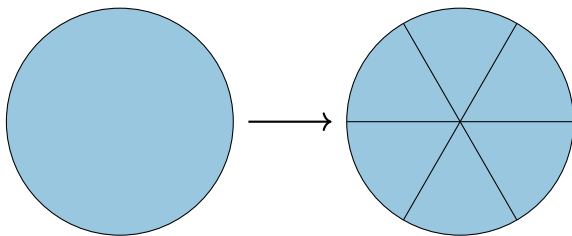
- An arrow from 6 to 2 is labeled $:3$.
- An arrow from 9 to 3 is labeled $:3$.



$$\frac{12}{16} = \frac{3}{4}$$

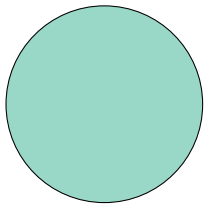
The equation shows the simplification of the fraction $\frac{12}{16}$ to $\frac{3}{4}$. Above the fraction bar, a pink arrow points from 12 to 3, with a pink $: 4$ above it. Below the fraction bar, a pink arrow points from 16 to 4, with a pink $: 4$ below it.

Cutting entire pieces

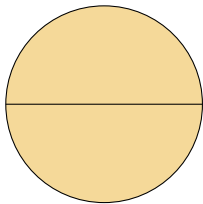


$$\frac{1}{1} = \frac{6}{6}$$

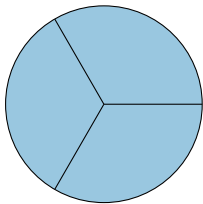
The diagram shows the fraction $\frac{1}{1}$ on the left and $\frac{6}{6}$ on the right, with an equals sign between them. A blue curved arrow points from the numerator 1 to the numerator 6, labeled with a blue $\times 6$. Another blue curved arrow points from the denominator 1 to the denominator 6, also labeled with a blue $\times 6$.



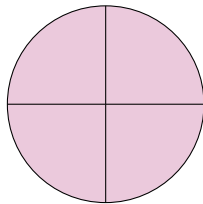
$$1 = \frac{1}{1}$$



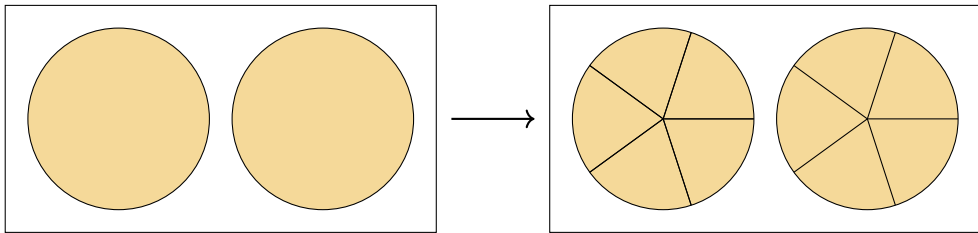
$$1 = \frac{2}{2}$$



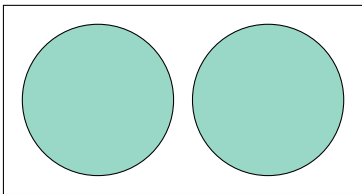
$$1 = \frac{3}{3}$$



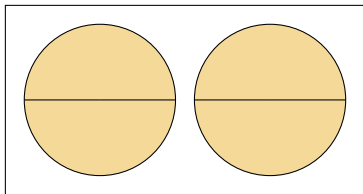
$$1 = \frac{4}{4}$$



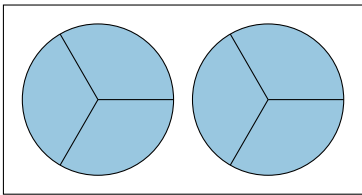
$$\frac{2}{1} \overset{\times 5}{\curvearrowright} \frac{10}{5}$$



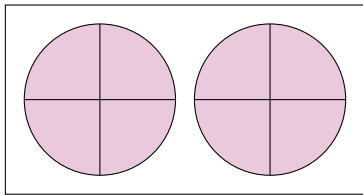
$$2 = \frac{2}{1}$$



$$2 = \frac{4}{2}$$

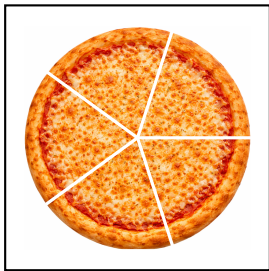
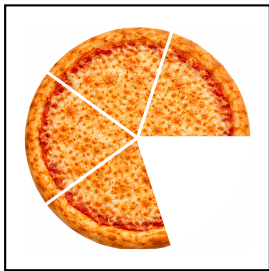


$$2 = \frac{6}{3}$$

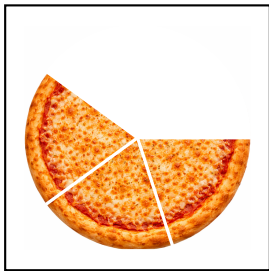
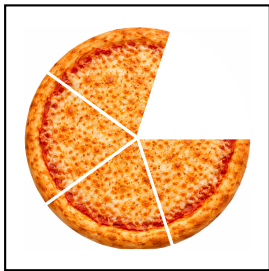


$$2 = \frac{8}{4}$$

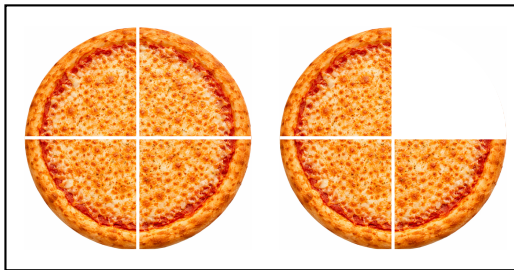
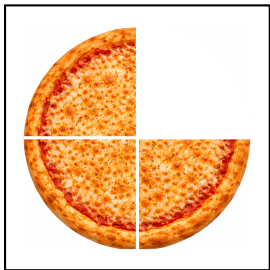
Comparing equal slices



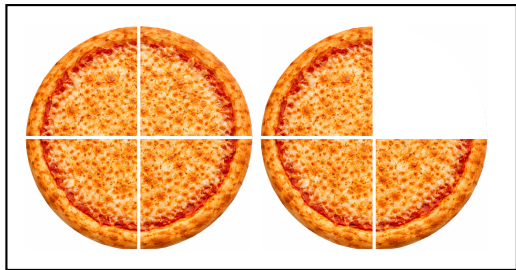
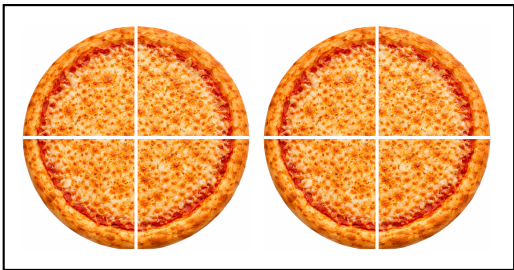
$$\frac{4}{5} < \frac{5}{5}$$



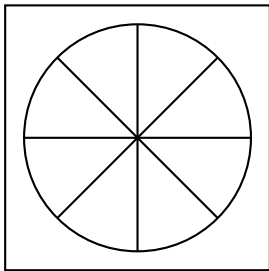
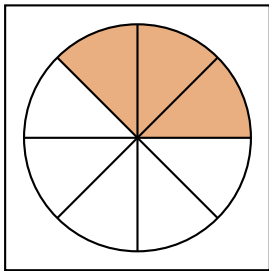
$$\frac{4}{5} > \frac{3}{5}$$



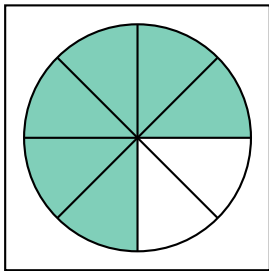
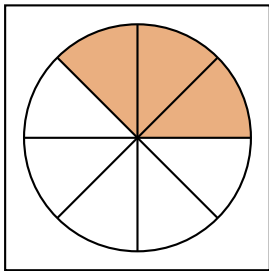
$$\frac{3}{4} < \frac{7}{4}$$



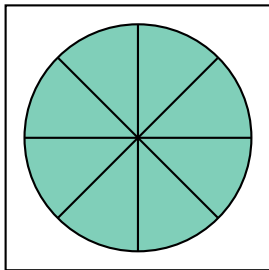
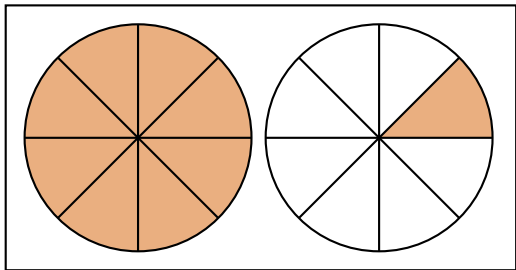
$$\frac{8}{4} > \frac{7}{4}$$



$$\frac{3}{8} > \frac{0}{8}$$

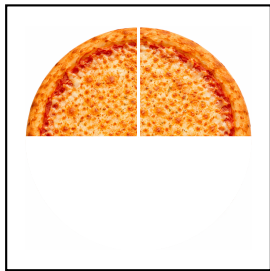
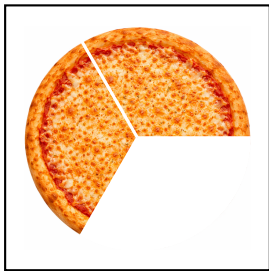


$$\frac{3}{8} < \frac{6}{8}$$

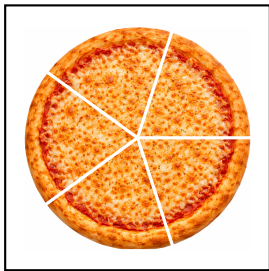
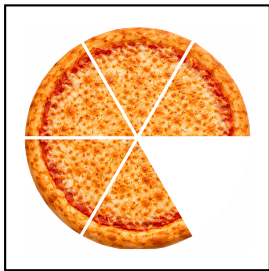


$$\frac{9}{8} > \frac{8}{8}$$

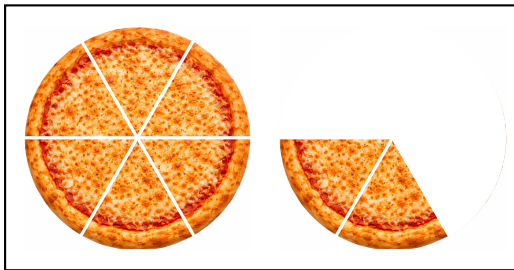
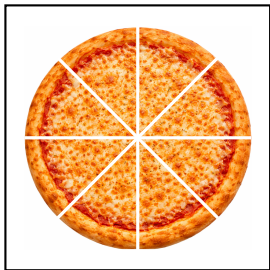
Comparing slices of different sizes



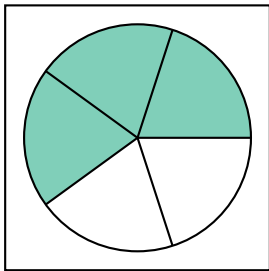
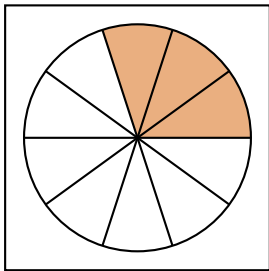
$$\frac{2}{3} > \frac{2}{4}$$



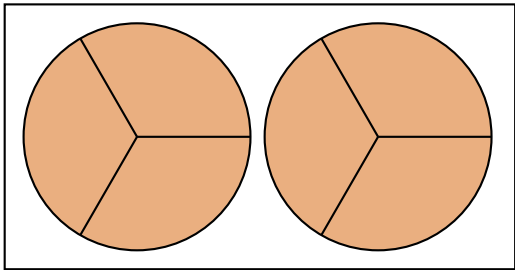
$$\frac{5}{6} < \frac{5}{5}$$



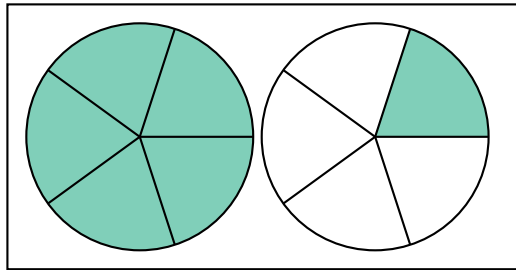
$$\frac{8}{8} < \frac{8}{6}$$



$$\frac{3}{10} > \frac{3}{5}$$



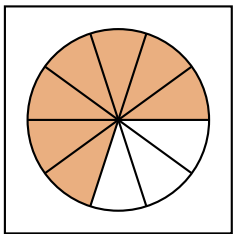
$$\frac{6}{3}$$



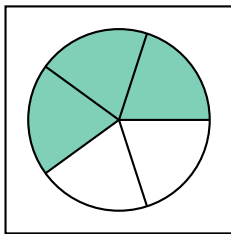
$$\frac{6}{5}$$

$$\frac{6}{3} < \frac{6}{5}$$

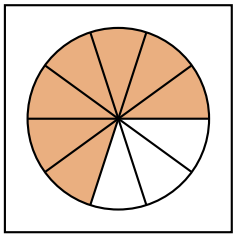
Comparing general amounts



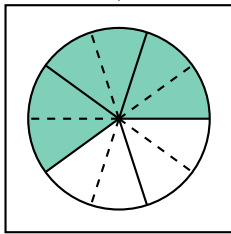
$$\frac{7}{10}$$



$$\downarrow \frac{3}{5}$$

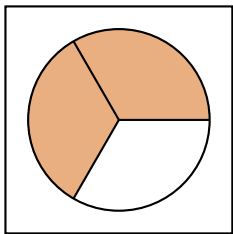


$$\frac{7}{10}$$

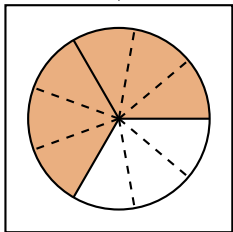


$$\frac{6}{10}$$

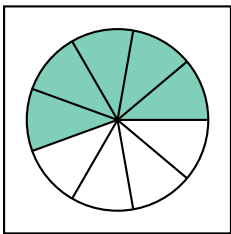
$$\frac{7}{10} > \frac{3}{5}$$



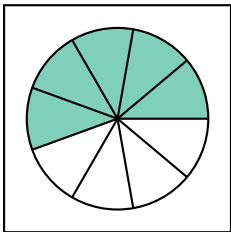
$\downarrow$ $\frac{2}{3}$



$\frac{4}{6}$



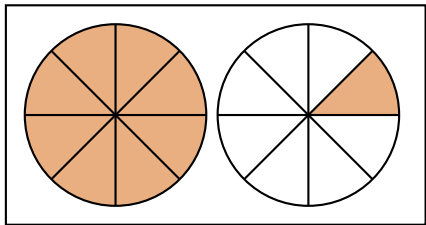
$\frac{5}{9}$

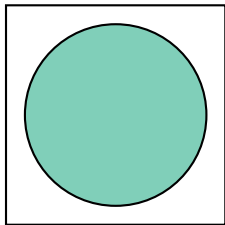


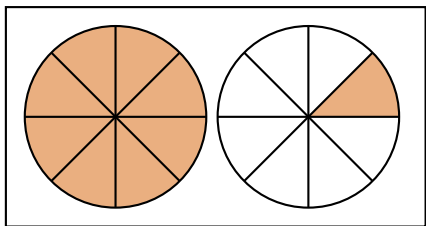
$\frac{4}{9}$

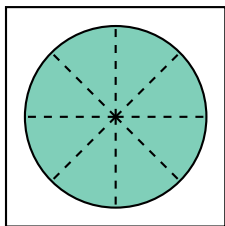
$$\frac{2}{3} > \frac{5}{9}$$

Comparing to entire pieces



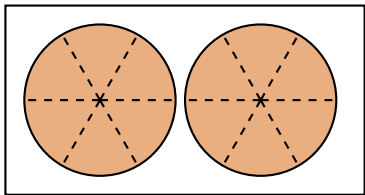
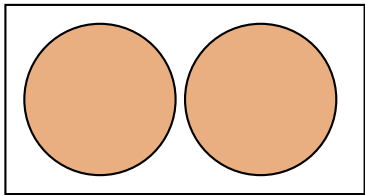
$$\frac{9}{8}$$


$$\downarrow 1$$


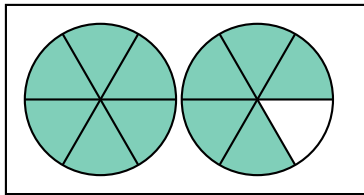
$$\frac{9}{8}$$


$$\frac{8}{8}$$

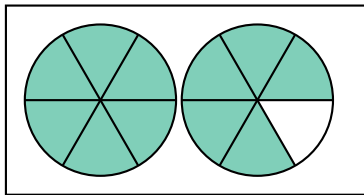
$$\frac{9}{8} > 1$$



$$\frac{12}{6}$$



$$\frac{11}{6}$$

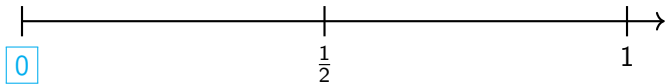
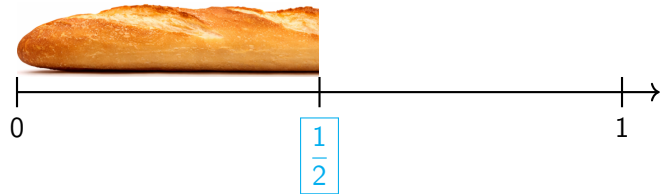


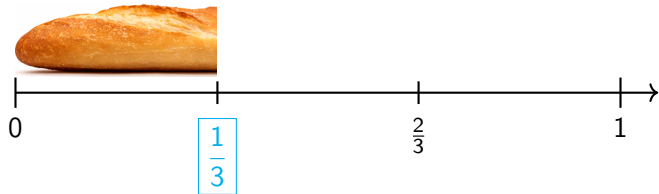
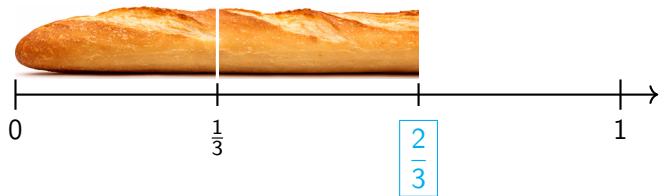
$$\frac{11}{6}$$

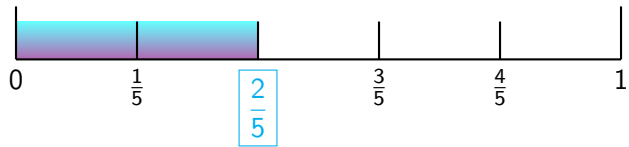
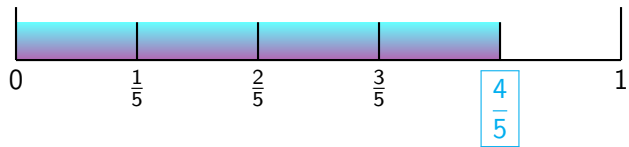
$$2 > \frac{11}{6}$$

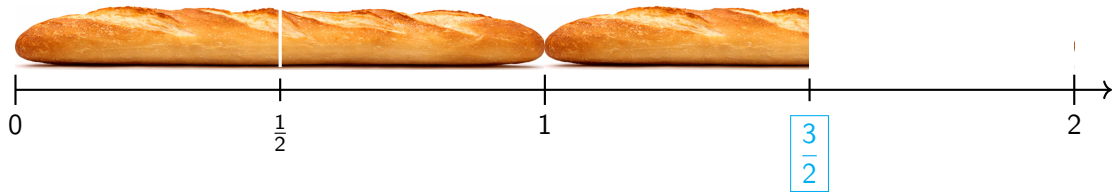
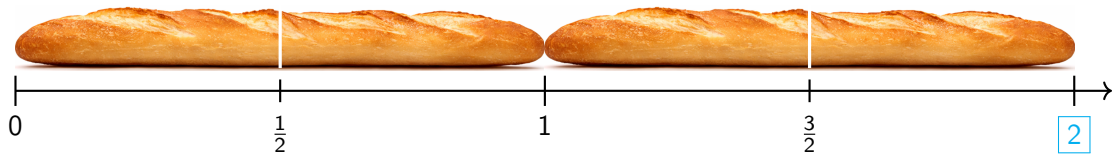
Fractions on the number line

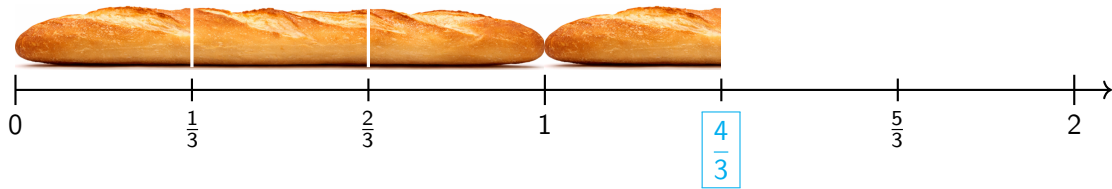
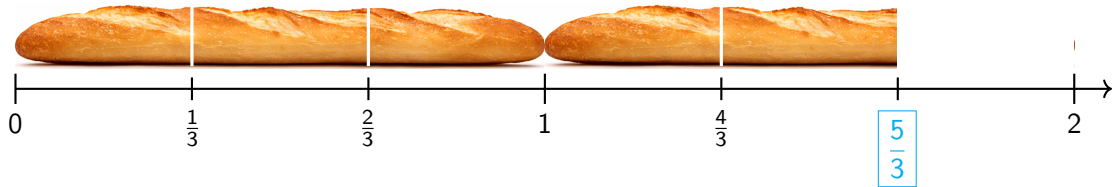


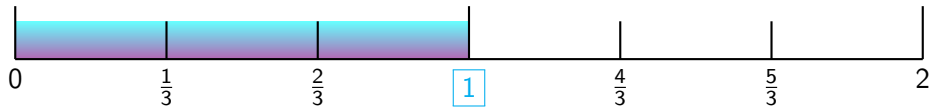
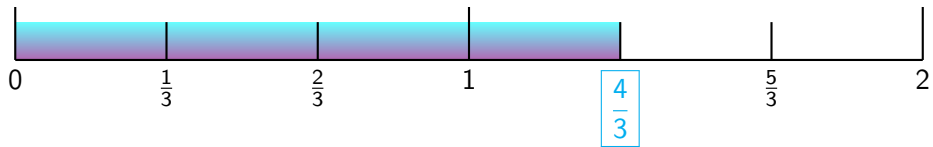
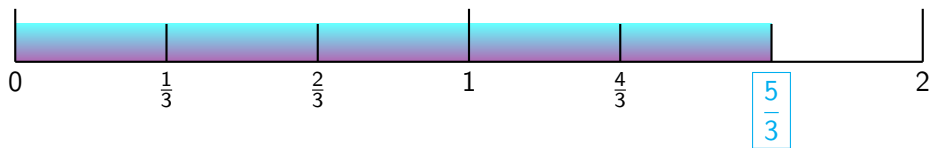


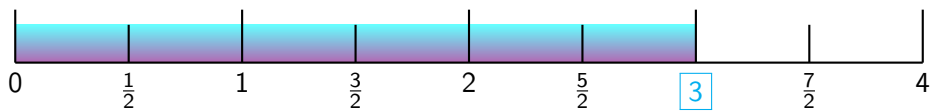
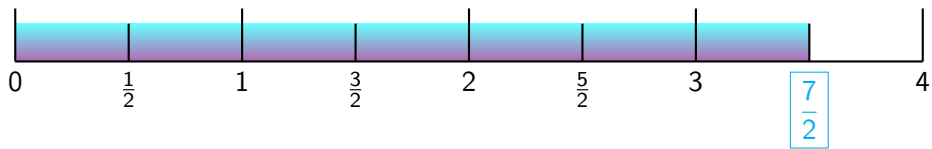




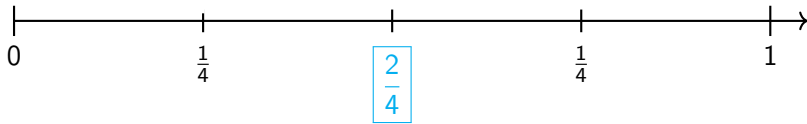
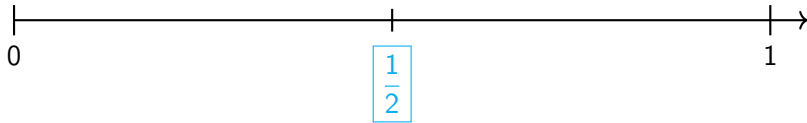




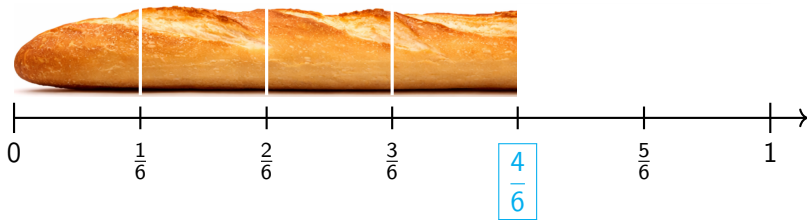




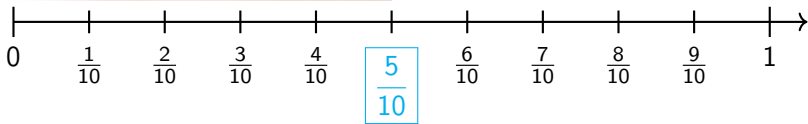
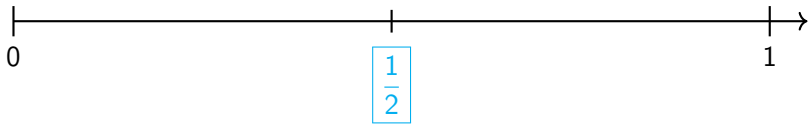
Equivalent fractions on the number line



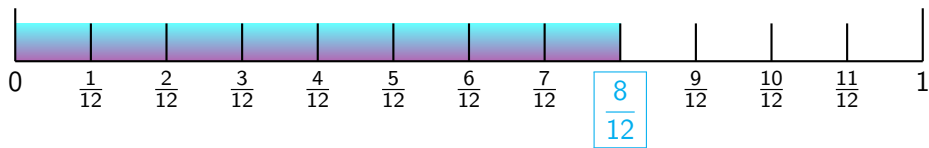
$$\begin{array}{ccc} & \times 2 & \\ \frac{1}{2} & \xrightarrow{\quad} & \frac{2}{4} \\ & \times 2 & \end{array}$$



$$\frac{2}{3} \xrightarrow{\times 2} \frac{4}{6}$$
$$\frac{2}{3} = \frac{4}{6}$$
$$\frac{4}{6} \xrightarrow{\times 2}$$

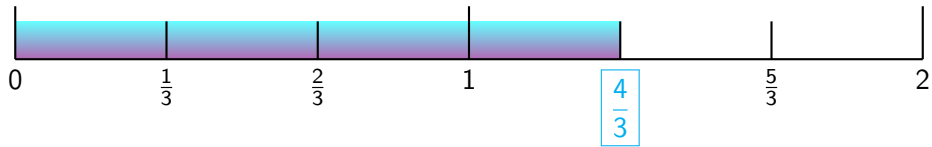
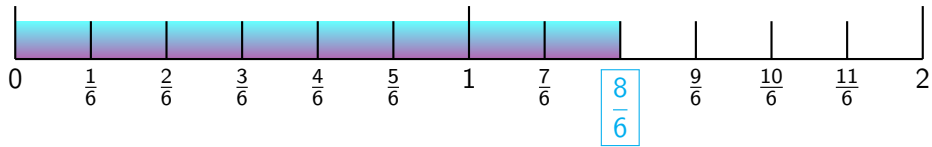


$$\frac{1}{2} \overset{\times 5}{=} \frac{5}{10}$$



$$\frac{2}{3} = \frac{8}{12}$$

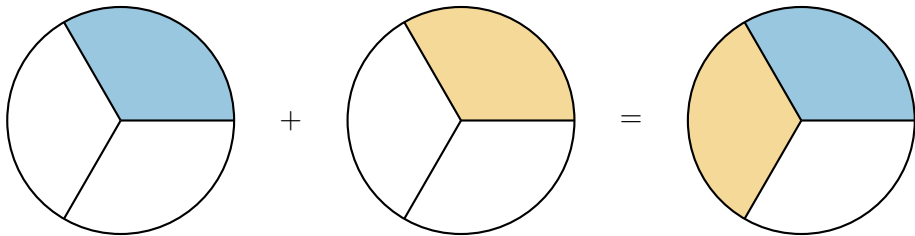
The equation is annotated with two curved arrows pointing from the left fraction to the right fraction. The top arrow is labeled $\times 4$ and points from the numerator 2 to 8. The bottom arrow is labeled $\times 4$ and points from the denominator 3 to 12.



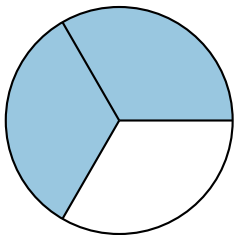
$$\frac{8}{6} = \frac{4}{3}$$

The diagram shows the simplification of the fraction $\frac{8}{6}$ to $\frac{4}{3}$. Two curved arrows indicate the process: the top arrow points from 8 to 4 with the label $: 2$, and the bottom arrow points from 6 to 3 with the label $: 2$.

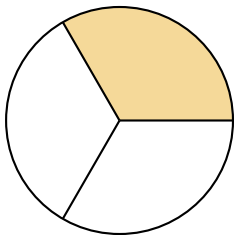
Collecting equal slices



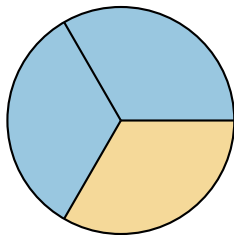
$$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$



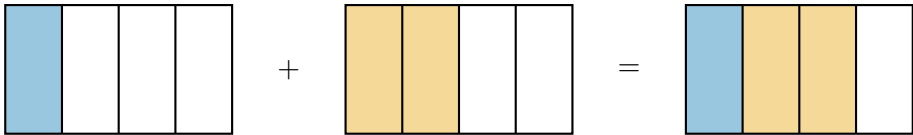
+



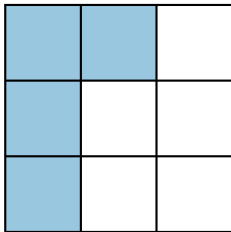
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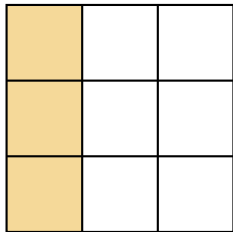
$$\frac{2}{3} + \frac{1}{3} = \frac{3}{3}$$



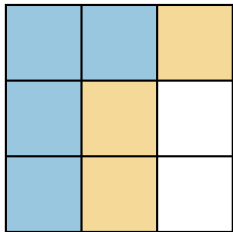
$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$



+

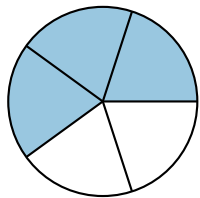


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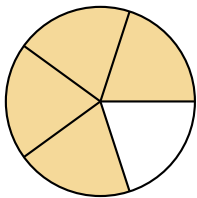


$$\frac{4}{9} + \frac{3}{9} = \frac{7}{9}$$

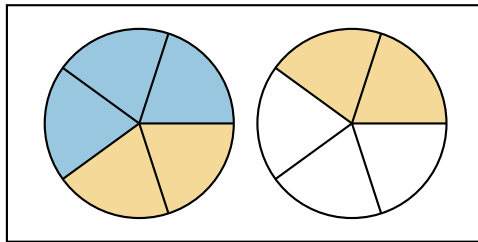
Collecting many equal slices



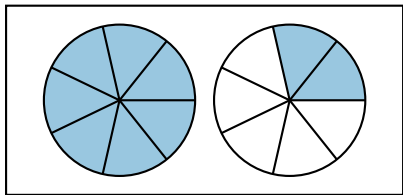
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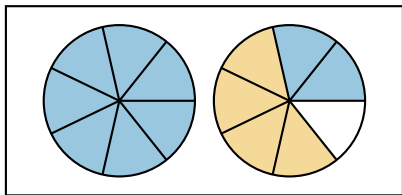
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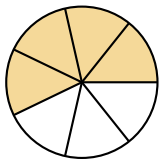
$$\frac{3}{5} + \frac{4}{5} = \frac{7}{5}$$



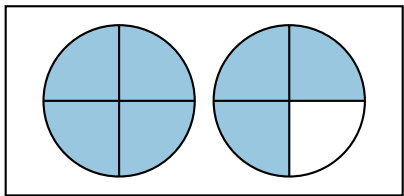
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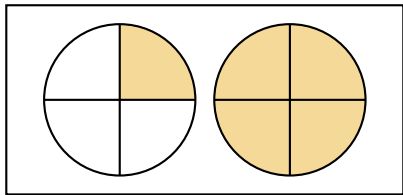
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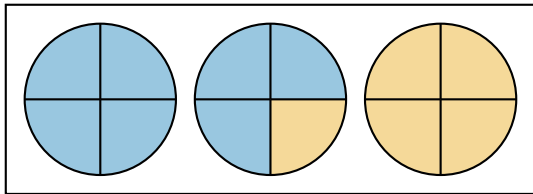
$$\frac{9}{7} + \frac{4}{7} = \frac{13}{7}$$



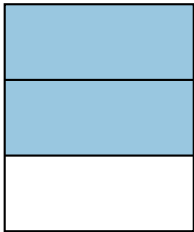
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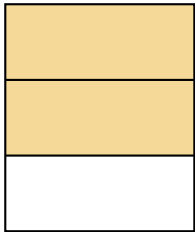
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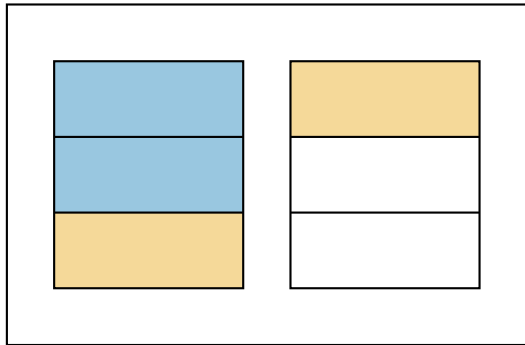
$$\frac{7}{4} + \frac{5}{4} = \frac{12}{4}$$



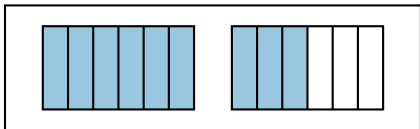
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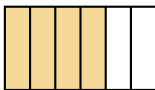
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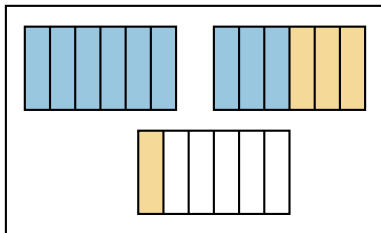
$$\frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$



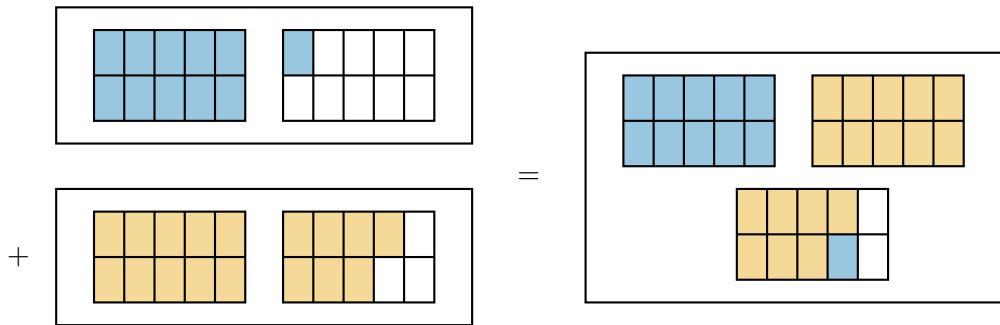
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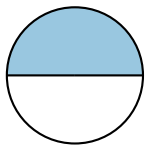


$$\frac{9}{6} + \frac{4}{6} = \frac{13}{6}$$

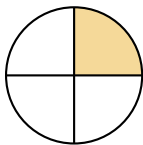


$$\frac{11}{10} + \frac{17}{10} = \frac{28}{10}$$

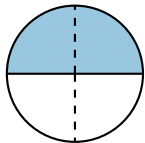
Collecting slices



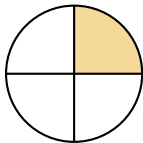
+



$$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$



+



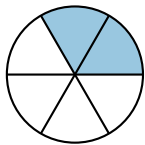
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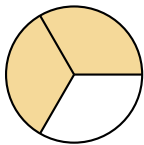
$$\frac{2}{4}$$

$$\frac{1}{4}$$

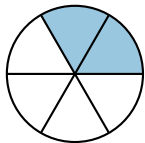
$$\frac{3}{4}$$



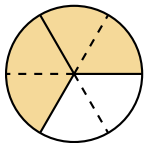
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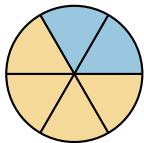
$$\frac{2}{6} + \frac{2}{3} = \frac{6}{6}$$



+



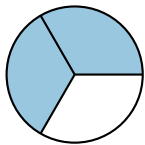
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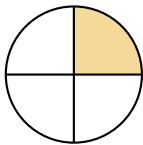
$$\frac{2}{6}$$

$$\frac{4}{6}$$

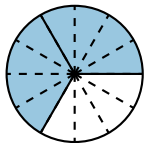
$$\frac{6}{6}$$



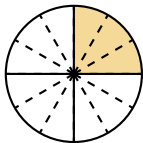
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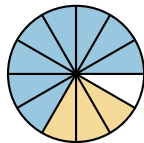
$$\frac{2}{3} + \frac{1}{4} = \frac{11}{12}$$



+



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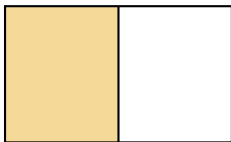
$$\frac{8}{12}$$

$$\frac{3}{12}$$

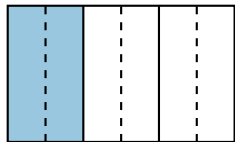
$$\frac{11}{12}$$



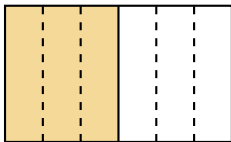
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$$\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$



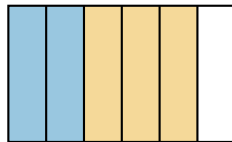
$$\frac{2}{6}$$



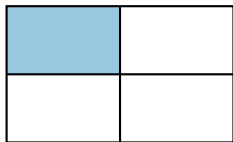
$$\frac{3}{6}$$

+

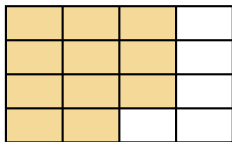
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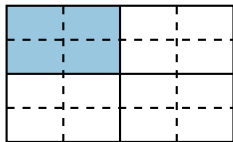
$$\frac{5}{6}$$



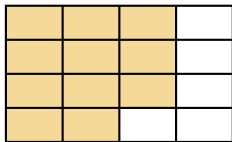
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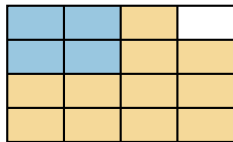
$$\frac{1}{4} + \frac{11}{16} = \frac{15}{16}$$



+



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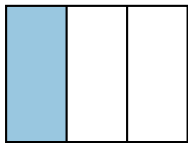


$$\frac{4}{16}$$

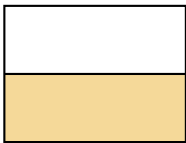
$$\frac{11}{16}$$

$$\frac{15}{16}$$

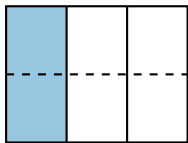
Collecting horizontal and vertical slices



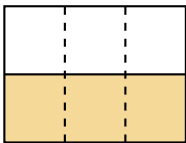
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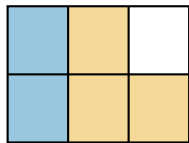
$$\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$



+



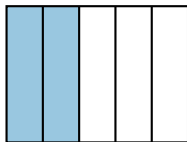
=



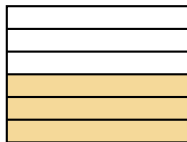
$$\frac{2}{6}$$

$$\frac{3}{6}$$

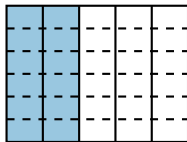
$$\frac{5}{6}$$



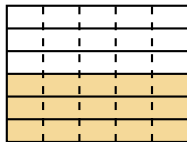
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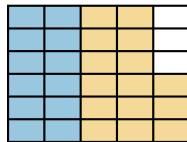
$$\frac{2}{5} + \frac{3}{6} = \frac{27}{30}$$



+



=

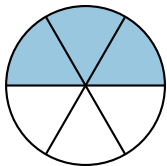


$$\frac{12}{30}$$

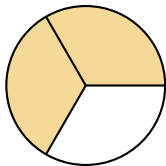
$$\frac{15}{30}$$

$$\frac{27}{30}$$

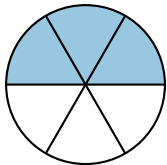
Collecting many slices



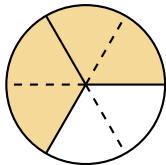
+



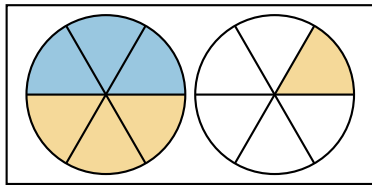
$$\frac{3}{6} + \frac{2}{3} = \frac{7}{6}$$



+



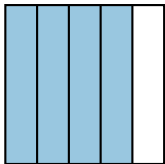
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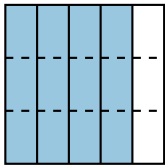
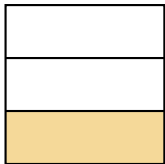
$$\frac{3}{6}$$

$$\frac{4}{6}$$

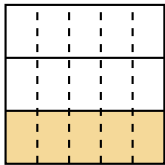
$$\frac{7}{6}$$



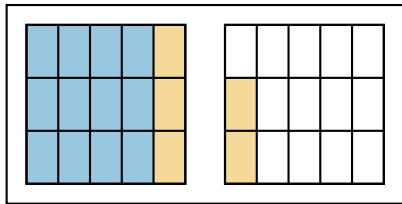
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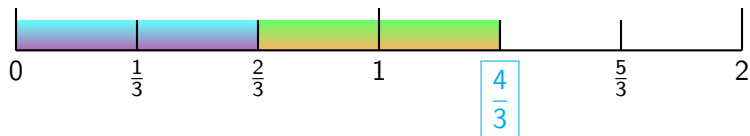
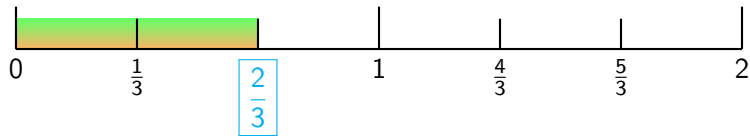
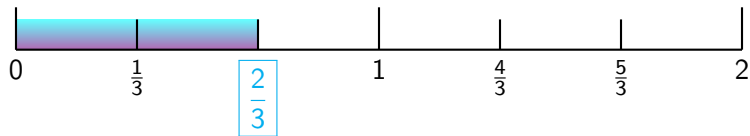
$$\frac{4}{5} + \frac{1}{3} = \frac{17}{15}$$

$$\frac{12}{15}$$

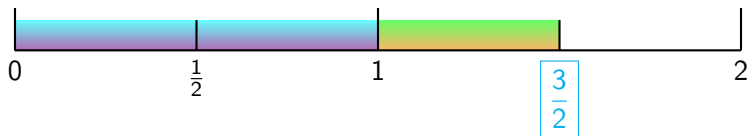
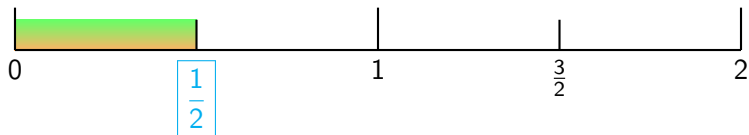
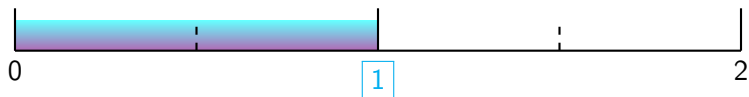
$$\frac{5}{15}$$

$$\frac{17}{15}$$

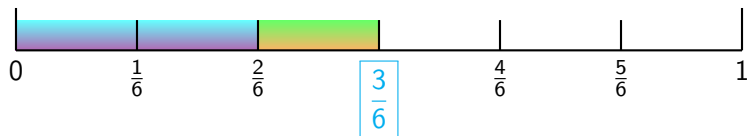
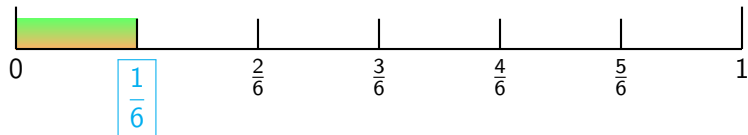
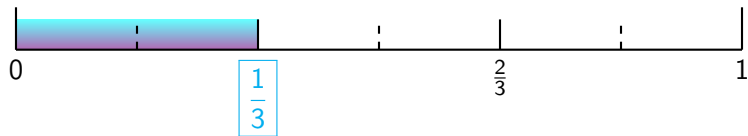
Addition on the number line



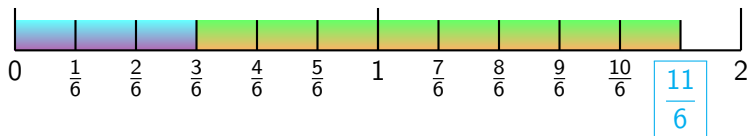
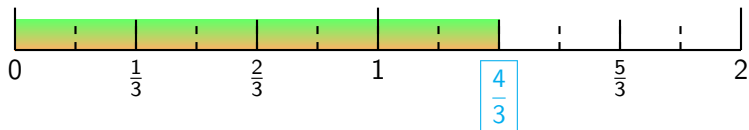
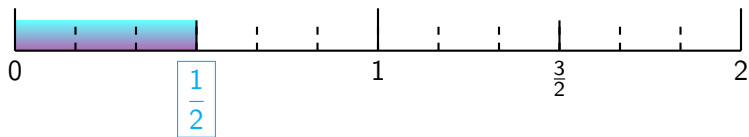
$$\frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$



$$1 + \frac{1}{2} = \frac{3}{2}$$

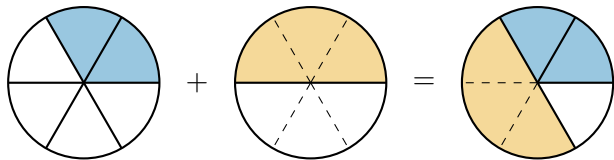


$$\frac{1}{3} + \frac{1}{6} = \frac{3}{6}$$

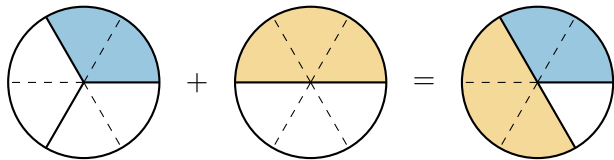


$$\frac{1}{2} + \frac{4}{3} = \frac{11}{6}$$

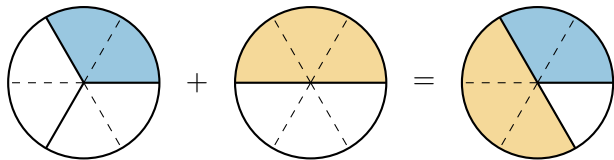
Addition with equivalent fractions



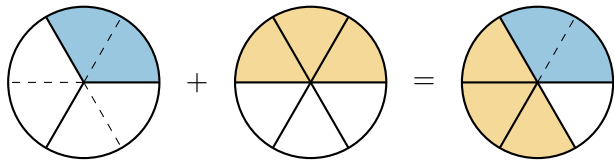
$$\frac{2}{6} + \frac{1}{2} = \frac{5}{6}$$



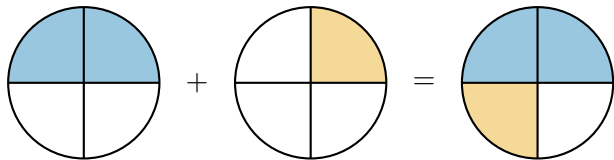
$$\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$



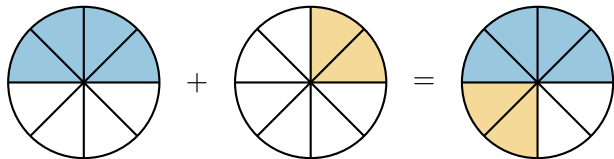
$$\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$



$$\frac{1}{3} + \frac{3}{6} = \frac{5}{6}$$

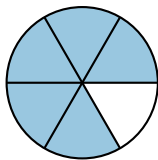


$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

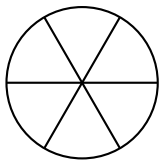


$$\frac{4}{8} + \frac{2}{8} = \frac{6}{8} = \frac{3}{4}$$

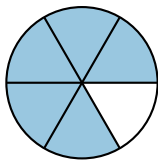
Adding zero



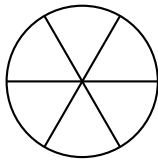
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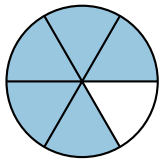
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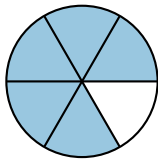
$$\frac{5}{6} + 0 = \frac{5}{6}$$



+

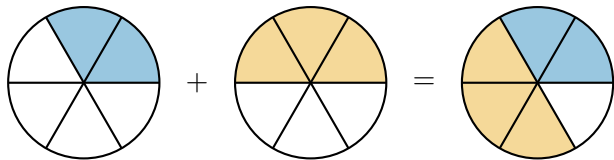


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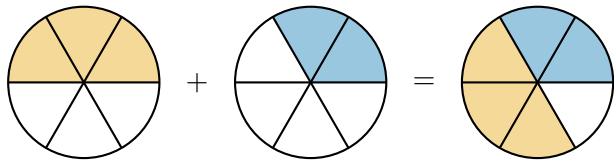


$$0 + \frac{5}{6} = \frac{5}{6}$$

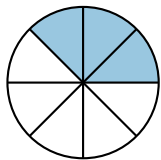
Commutativity



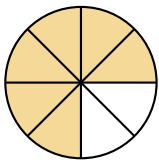
$$\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$$



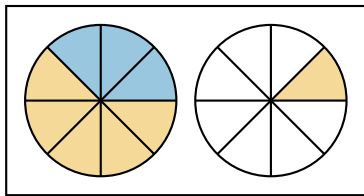
$$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$



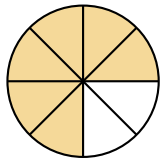
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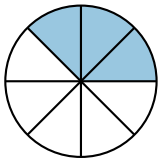
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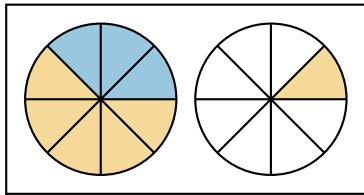
$$\frac{3}{8} + \frac{6}{8} = \frac{9}{8}$$



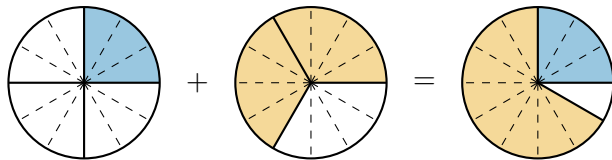
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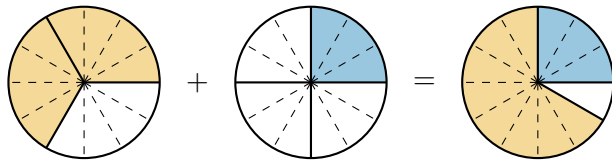
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$$\frac{6}{8} + \frac{3}{8} = \frac{9}{8}$$



$$\frac{1}{4} + \frac{2}{3} = \frac{11}{12}$$



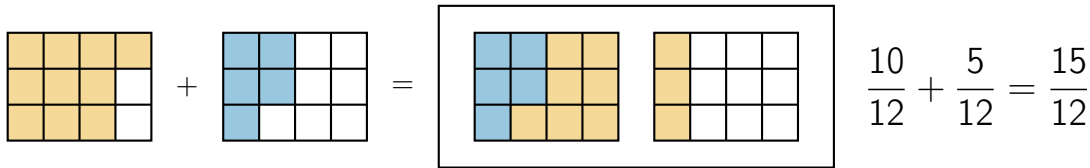
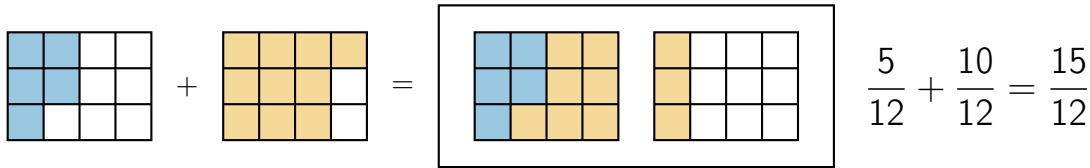
$$\frac{2}{3} + \frac{1}{4} = \frac{11}{12}$$

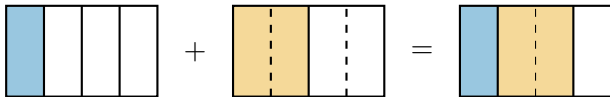


$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$



$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$



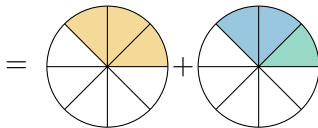
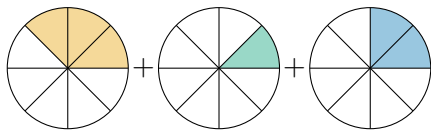
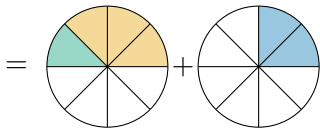
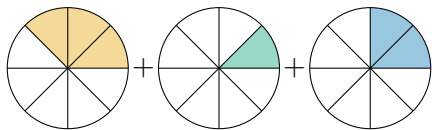


$$\frac{1}{2} + \frac{2}{4} = \frac{3}{4}$$



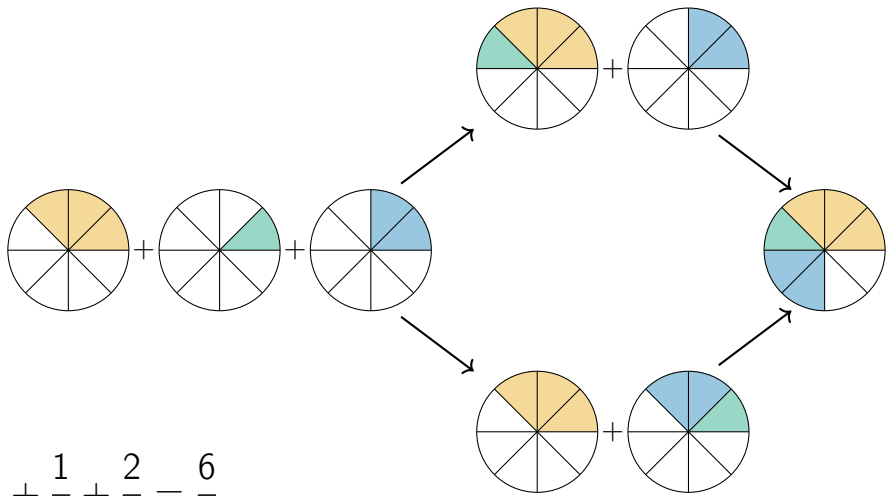
$$\frac{2}{4} + \frac{1}{2} = \frac{3}{4}$$

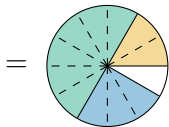
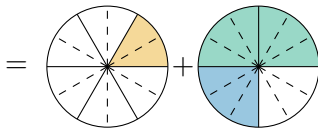
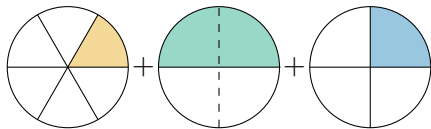
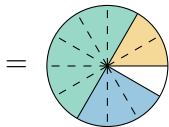
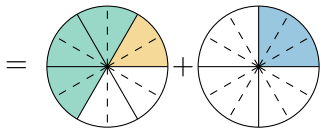
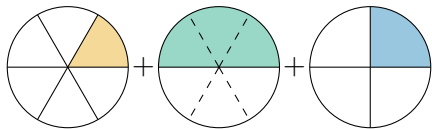
Associativity



$$\frac{3}{8} + \frac{1}{8} + \frac{2}{8} = \frac{6}{8}$$

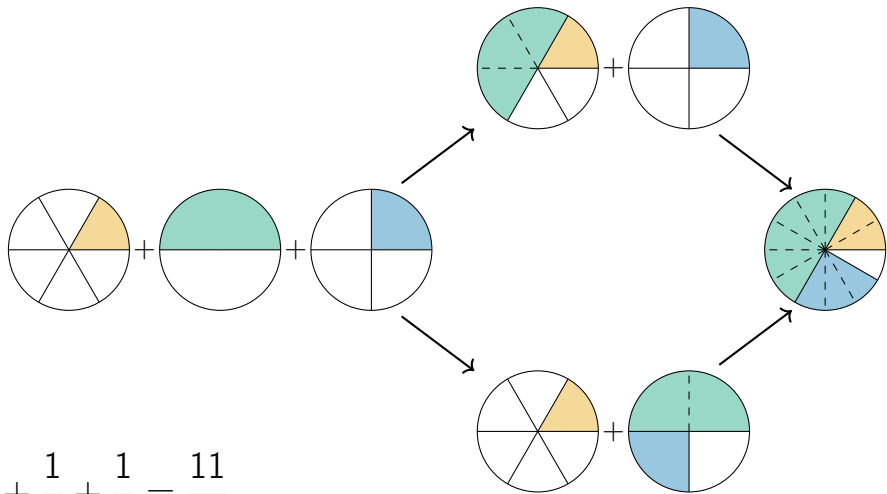
$$\frac{3}{8} + \frac{1}{8} + \frac{2}{8} = \frac{6}{8}$$



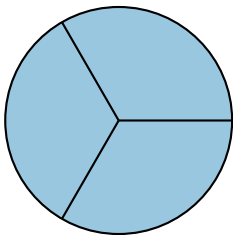


$$\frac{1}{6} + \frac{1}{2} + \frac{1}{4} = \frac{11}{12}$$

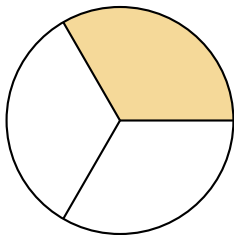
$$\frac{1}{6} + \frac{1}{2} + \frac{1}{4} = \frac{11}{12}$$



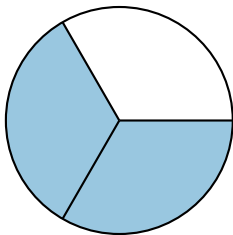
Removing equal slices



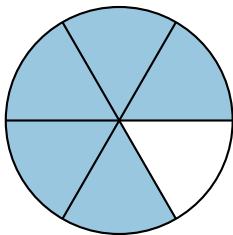
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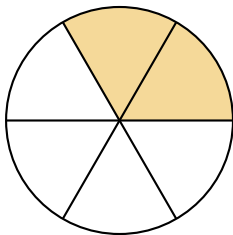
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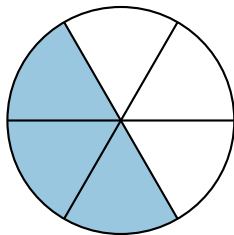
$$\frac{3}{3} - \frac{1}{3} = \frac{2}{3}$$



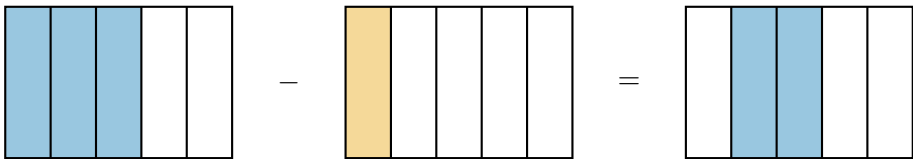
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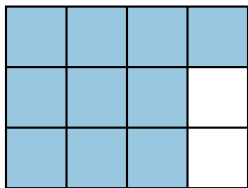
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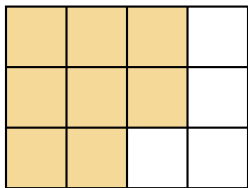
$$\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$$



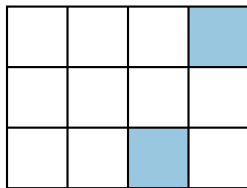
$$\frac{3}{5} - \frac{1}{5} = \frac{2}{5}$$



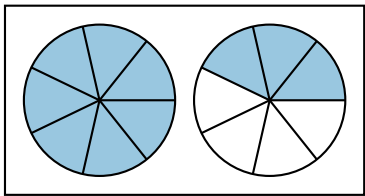
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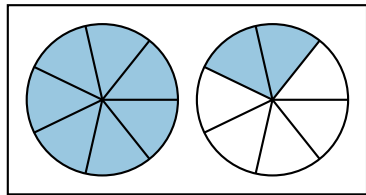
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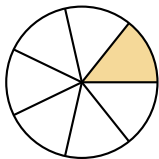
$$\frac{10}{12} - \frac{8}{12} = \frac{2}{12}$$



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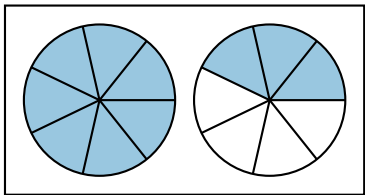


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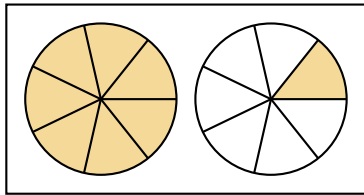


$$\frac{10}{7} - \frac{1}{7} = \frac{9}{7}$$

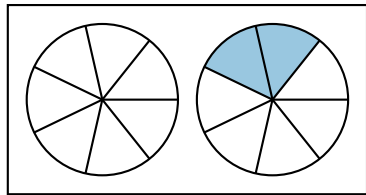
Removing many equal slices



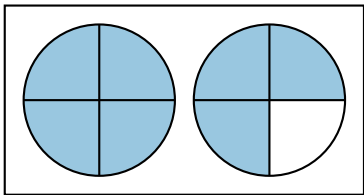
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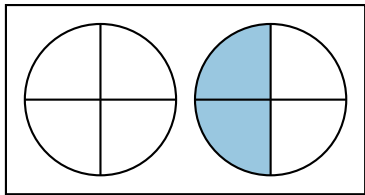
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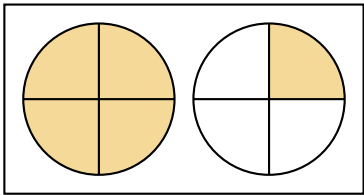
$$\frac{10}{7} - \frac{8}{7} = \frac{2}{7}$$



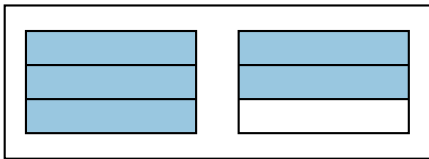
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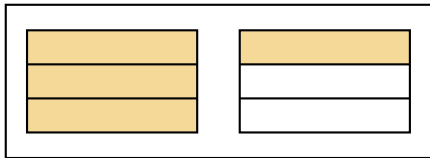
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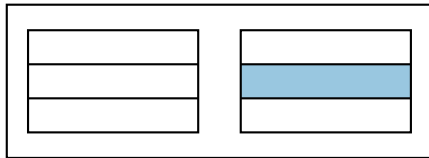
$$\frac{7}{4} - \frac{5}{4} = \frac{2}{4}$$



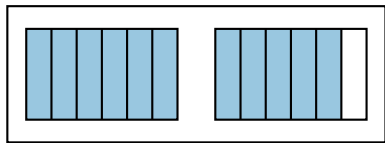
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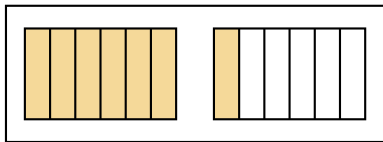
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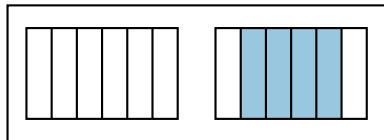
$$\frac{5}{3} - \frac{4}{3} = \frac{1}{3}$$



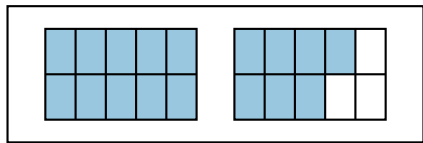
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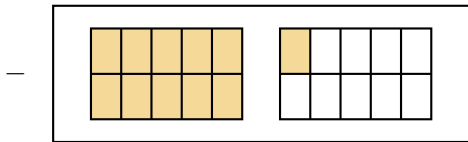
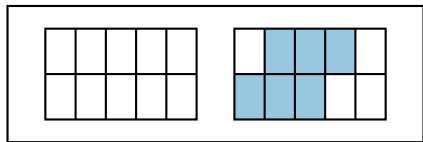
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$$\frac{11}{6} - \frac{7}{6} = \frac{4}{6}$$

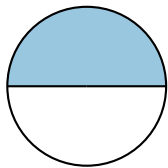


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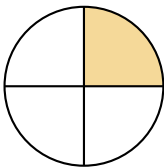


$$\frac{17}{10} - \frac{11}{10} = \frac{6}{10}$$

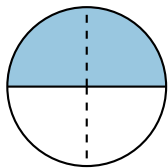
Removing slices



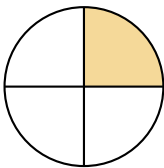
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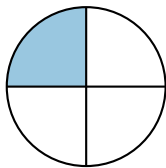
$$\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$



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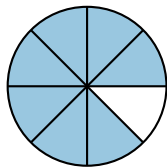
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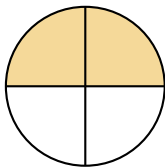
$$\frac{2}{4}$$

$$\frac{1}{4}$$

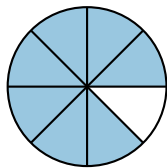
$$\frac{1}{4}$$



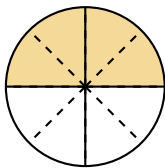
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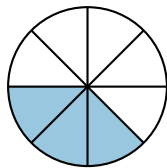
$$\frac{7}{8} - \frac{2}{4} = \frac{3}{8}$$



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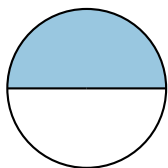
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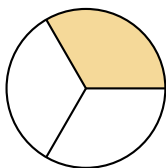
$$\frac{7}{8}$$

$$\frac{4}{8}$$

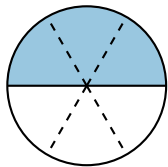
$$\frac{3}{8}$$



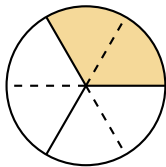
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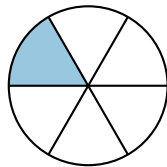
$$\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$



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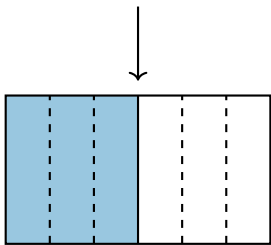
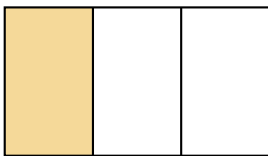
$$\frac{3}{6}$$

$$\frac{2}{6}$$

$$\frac{1}{6}$$

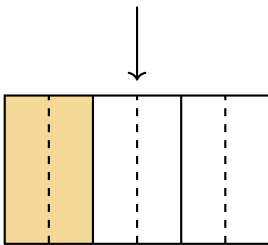


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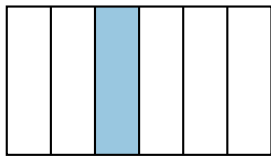
$$\frac{3}{6}$$

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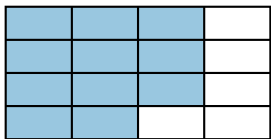
$$\frac{2}{6}$$

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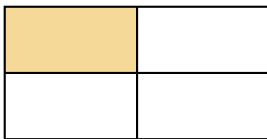


$$\frac{1}{6}$$

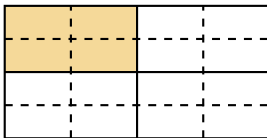
$$\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$



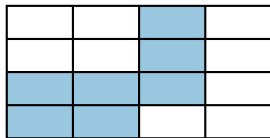
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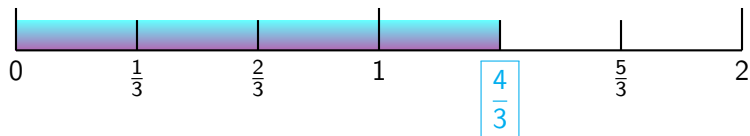
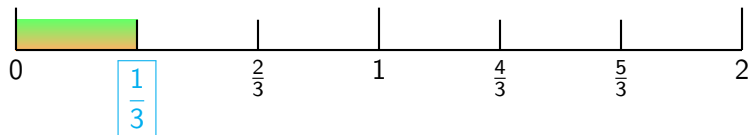
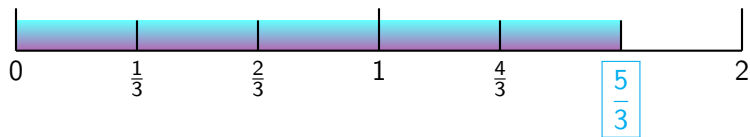
$$\frac{11}{16} - \frac{1}{4} = \frac{7}{16}$$

$$\frac{11}{16}$$

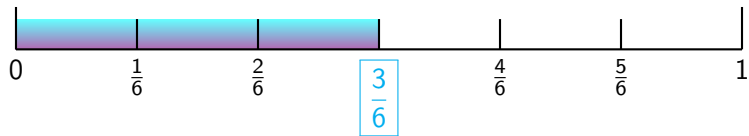
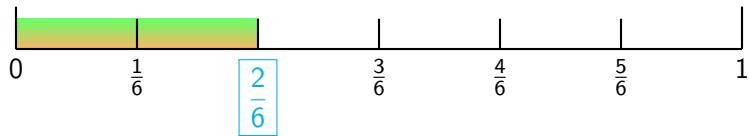
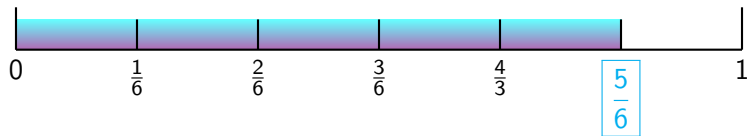
$$\frac{4}{16}$$

$$\frac{7}{16}$$

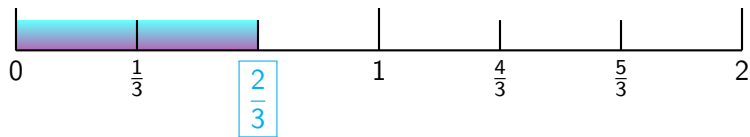
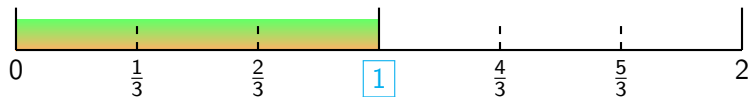
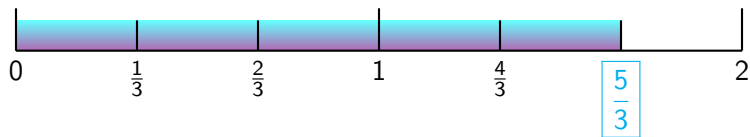
Subtraction on the number line



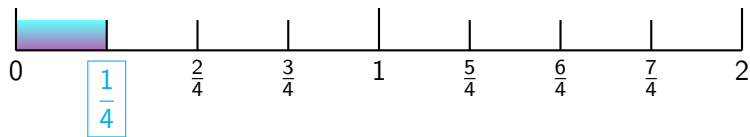
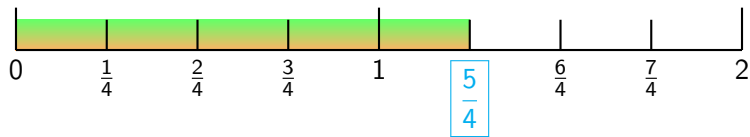
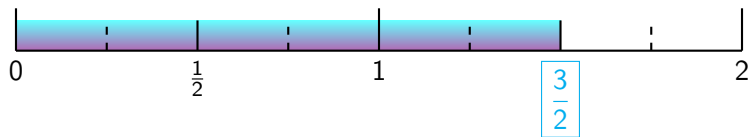
$$\frac{5}{3} - \frac{1}{3} = \frac{4}{3}$$



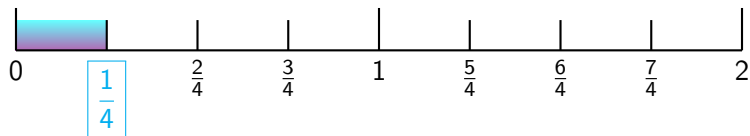
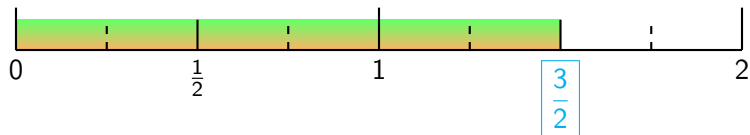
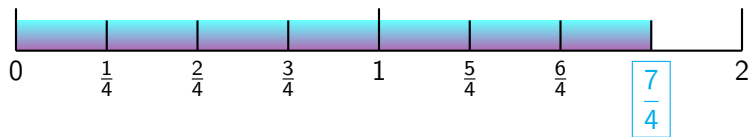
$$\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$$



$$\frac{5}{3} - 1 = \frac{2}{3}$$

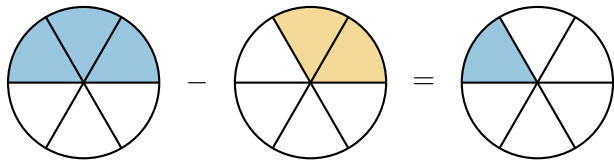


$$\frac{3}{2} - \frac{5}{4} = \frac{1}{4}$$

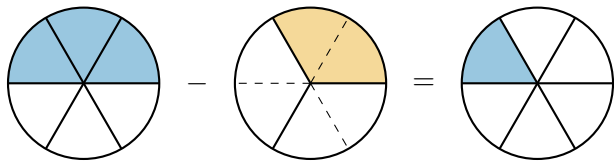


$$\frac{7}{4} - \frac{3}{2} = \frac{1}{4}$$

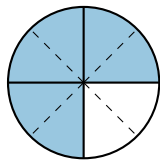
Subtraction with equivalent fractions



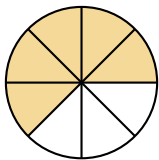
$$\frac{3}{6} - \frac{2}{6} = \frac{1}{6}$$



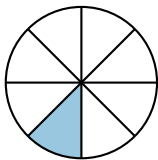
$$\frac{3}{6} - \frac{1}{3} = \frac{1}{6}$$



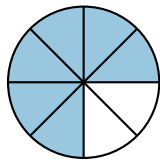
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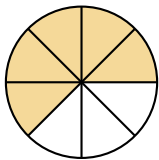
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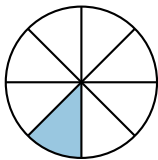
$$\frac{3}{4} - \frac{5}{8} = \frac{1}{8}$$



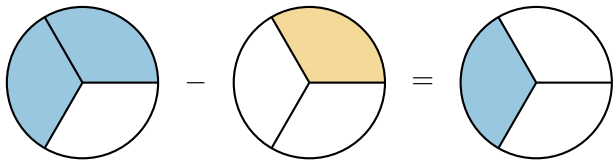
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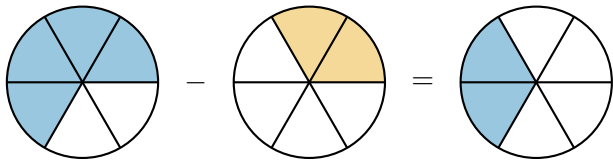
=



$$\frac{6}{8} - \frac{5}{8} = \frac{1}{8}$$

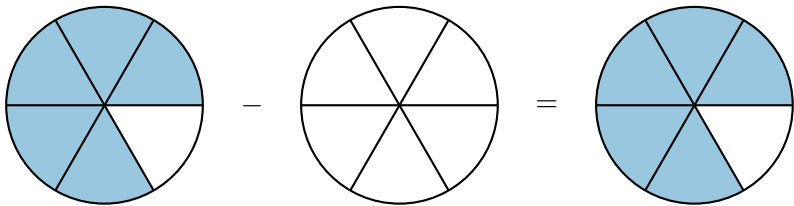


$$\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$



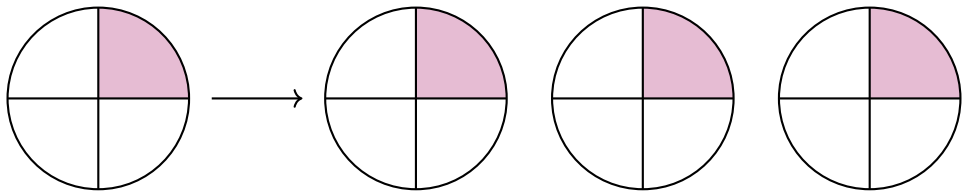
$$\frac{4}{6} - \frac{2}{6} = \frac{1}{3}$$

Subtracting zero

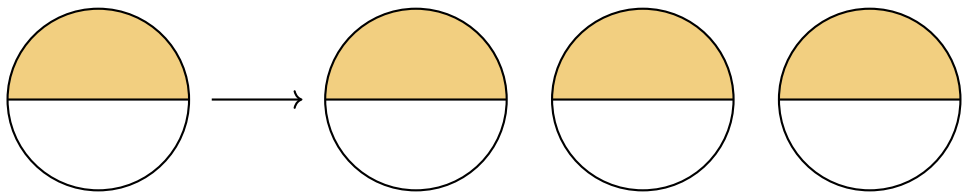


$$\frac{5}{6} - 0 = \frac{5}{6}$$

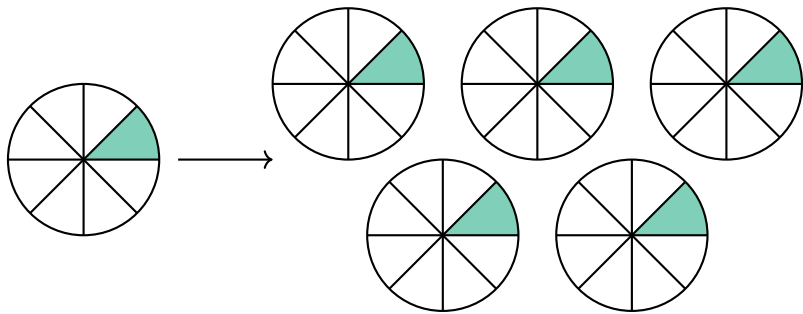
Integer $\times$ Fraction



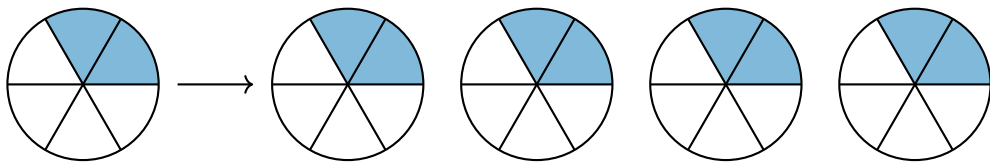
$$3 \times \frac{1}{4} = \frac{3}{4}$$



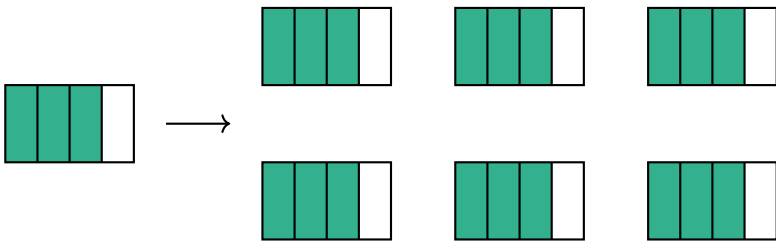
$$3 \times \frac{1}{2} = \frac{3}{2}$$



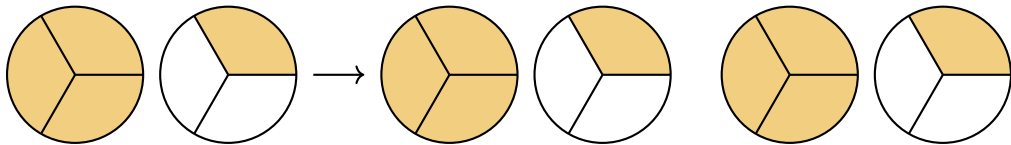
$$5 \times \frac{1}{8} = \frac{5}{8}$$



$$4 \times \frac{2}{6} = \frac{8}{6}$$

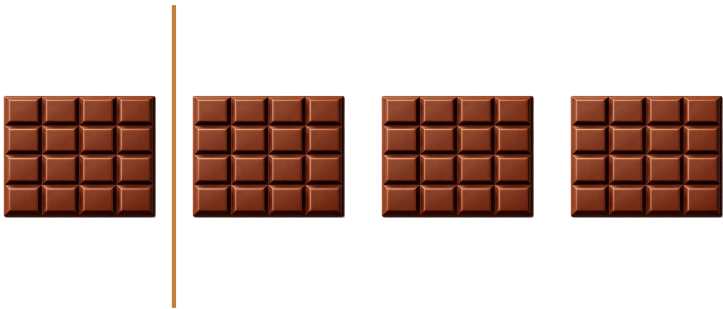


$$6 \times \frac{3}{4} = \frac{18}{4}$$

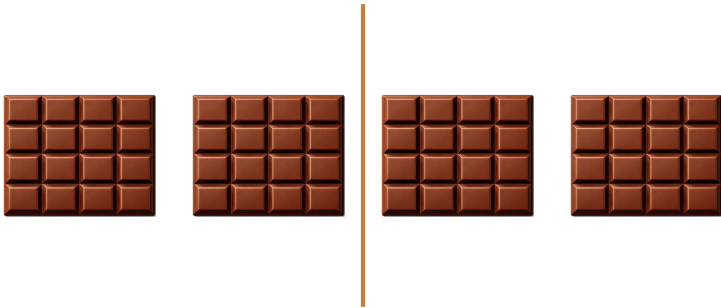


$$2 \times \frac{4}{3} = \frac{8}{3}$$

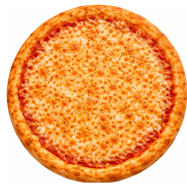
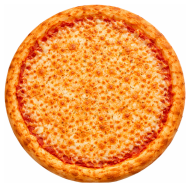
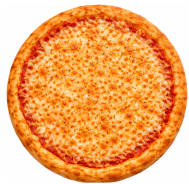
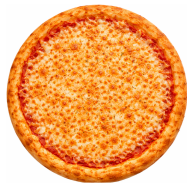
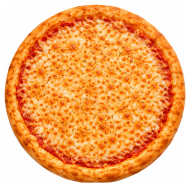
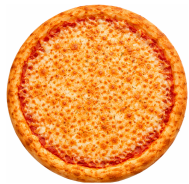
Integer $\times$ Unit fraction (with a partition)



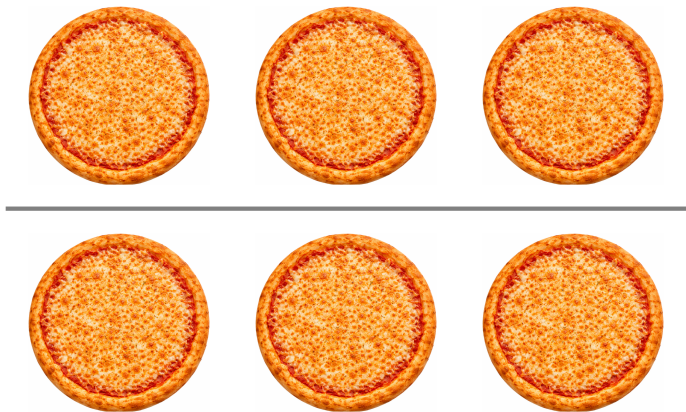
$$\frac{1}{4} \times 4 = 1$$



$$\frac{1}{2} \times 4 = 2$$

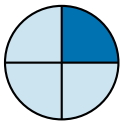
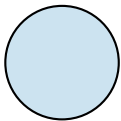


$$\frac{1}{3} \times 6 = 2$$

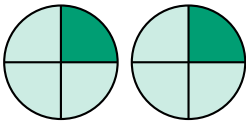
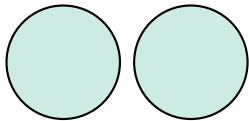


$$\frac{1}{2} \times 6 = 3$$

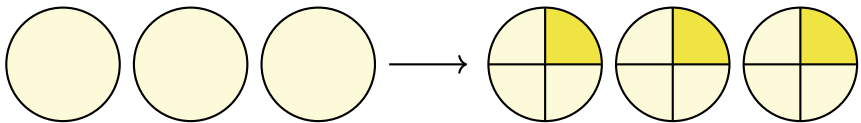
Integer $\times$ Unit fraction (in general)



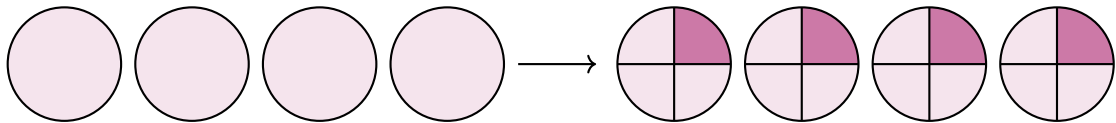
$$\frac{1}{4} \times 1 = \frac{1}{4}$$



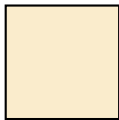
$$\frac{1}{4} \times 2 = \frac{2}{4}$$



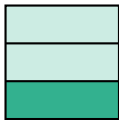
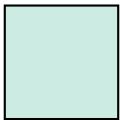
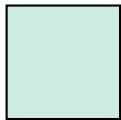
$$\frac{1}{4} \times 3 = \frac{3}{4}$$



$$\frac{1}{4} \times 4 = 1$$

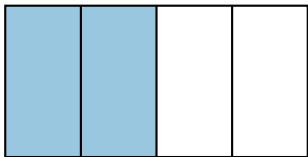


$$\frac{1}{3} \times 1 = \frac{1}{3}$$

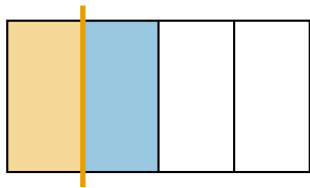
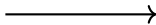


$$\frac{1}{3} \times 2 = \frac{2}{3}$$

Fraction $\times$ Unit fraction

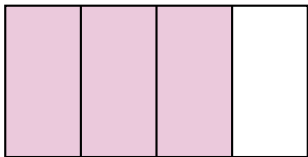


$$\frac{2}{4}$$

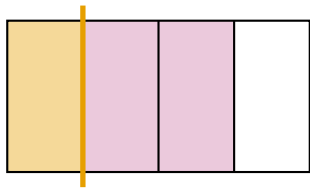


$$\frac{1}{2} \times \frac{2}{4}$$

$$\frac{1}{2} \times \frac{2}{4} = \frac{1}{4}$$

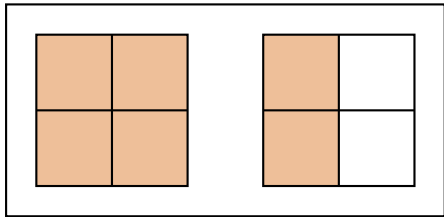


$$\frac{3}{4}$$

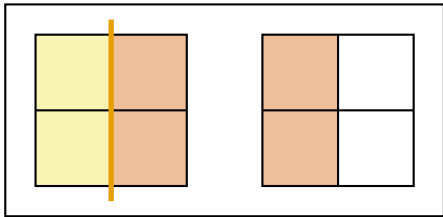


$$\frac{1}{3} \times \frac{3}{4}$$

$$\frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$$

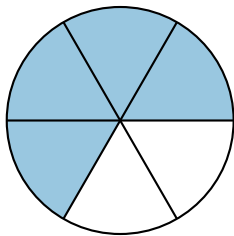


$$\frac{6}{4}$$

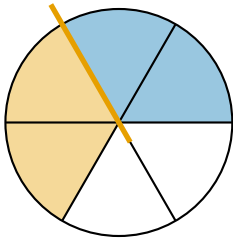


$$\frac{1}{3} \times \frac{3}{4}$$

$$\frac{1}{3} \times \frac{6}{4} = \frac{1}{2}$$

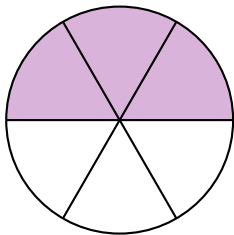


$$\frac{4}{6}$$

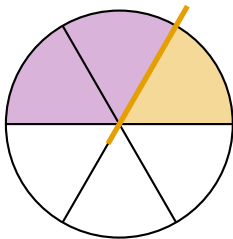


$$\frac{1}{2} \times \frac{4}{6}$$

$$\frac{1}{2} \times \frac{4}{6} = \frac{2}{6}$$

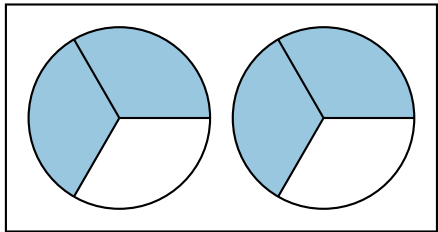


$$\frac{3}{6}$$

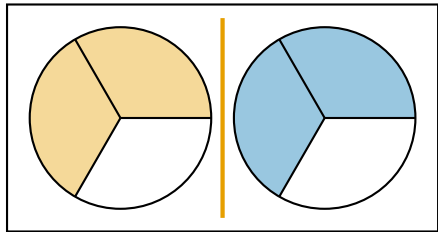


$$\frac{1}{3} \times \frac{3}{6}$$

$$\frac{1}{3} \times \frac{3}{6} = \frac{1}{6}$$

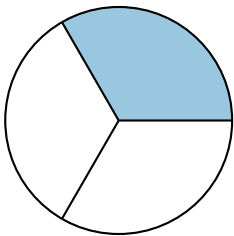


$$\frac{4}{3}$$

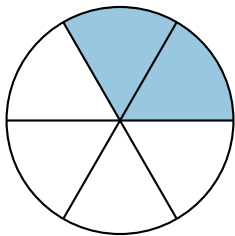


$$\frac{1}{2} \times \frac{4}{3}$$

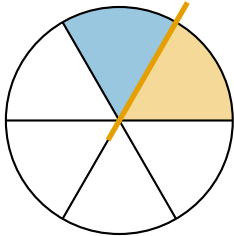
$$\frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$$



$$\frac{1}{3}$$

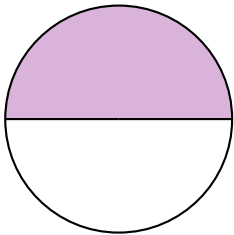


$$\frac{2}{6}$$

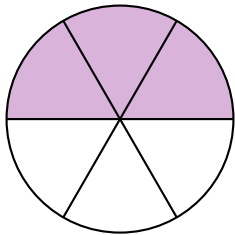


$$\frac{1}{2} \times \frac{2}{6}$$

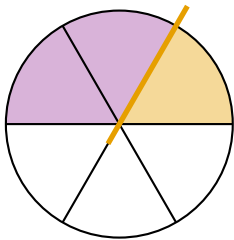
$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$



$$\frac{1}{2}$$



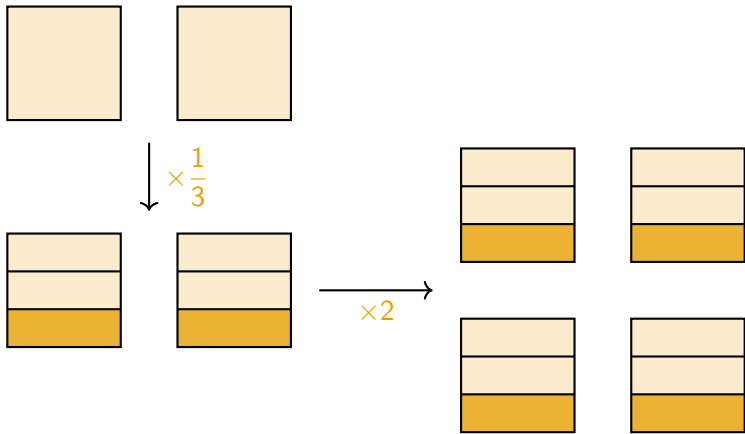
$$\frac{3}{6}$$



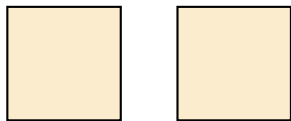
$$\frac{1}{3} \times \frac{3}{6}$$

$$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

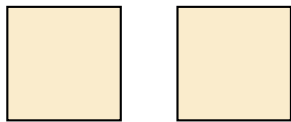
Integer $\times$ Fraction (two steps)



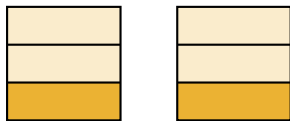
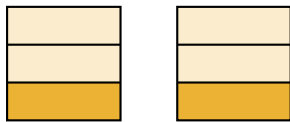
$$\frac{2}{3} \times 2 = \frac{4}{3}$$



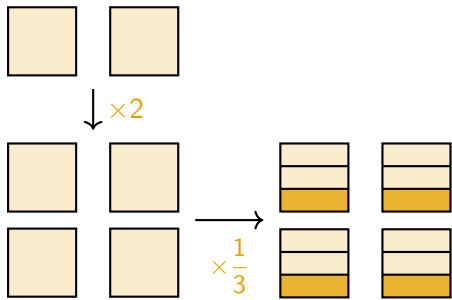
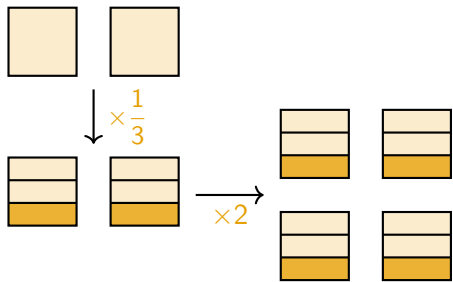
↓
×2



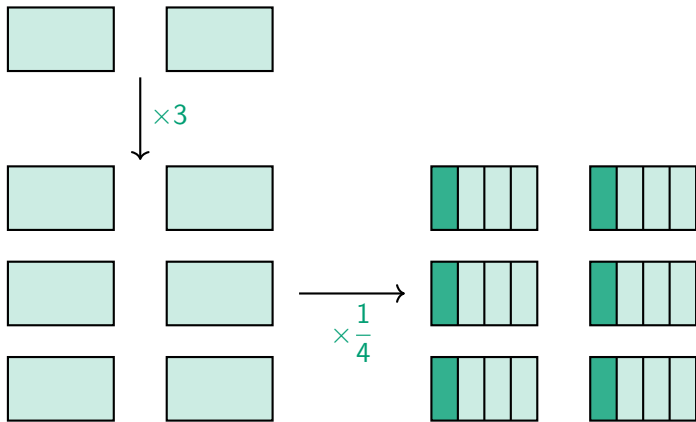
×
 $\frac{1}{3}$



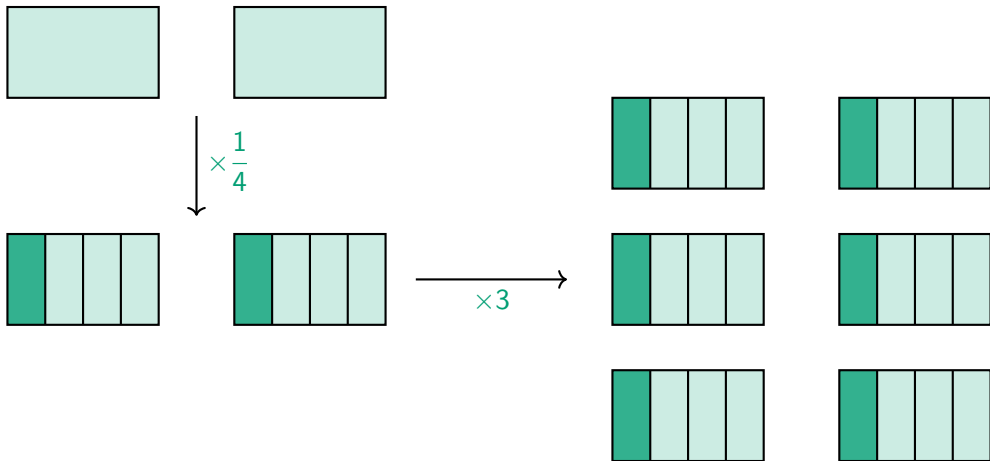
$$\frac{2}{3} \times 2 = \frac{4}{3}$$



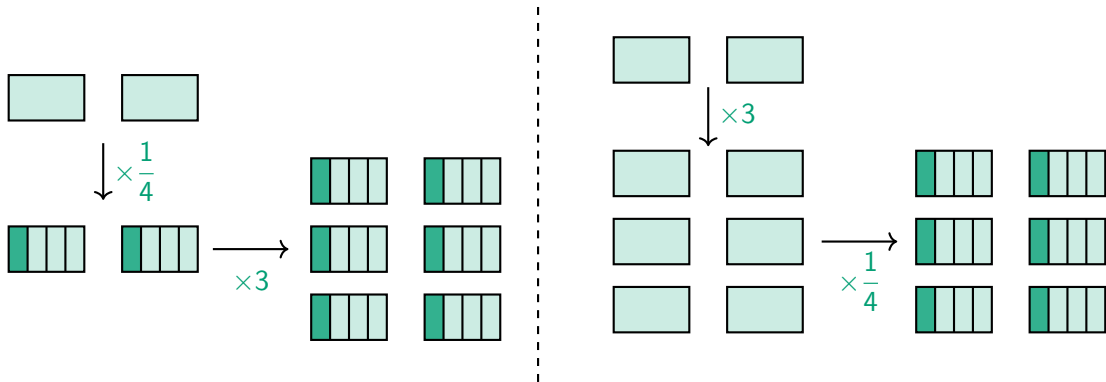
$$\frac{2}{3} \times 2 = \frac{4}{3}$$



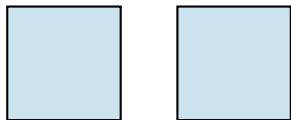
$$\frac{3}{4} \times 2 = \frac{6}{4}$$



$$\frac{3}{4} \times 2 = \frac{6}{4}$$



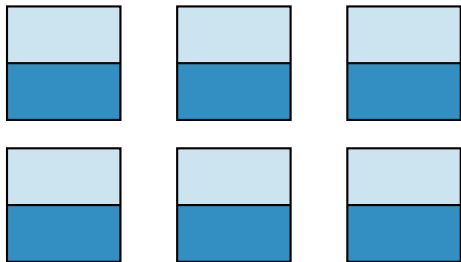
$$\frac{3}{4} \times 2 = \frac{6}{4}$$



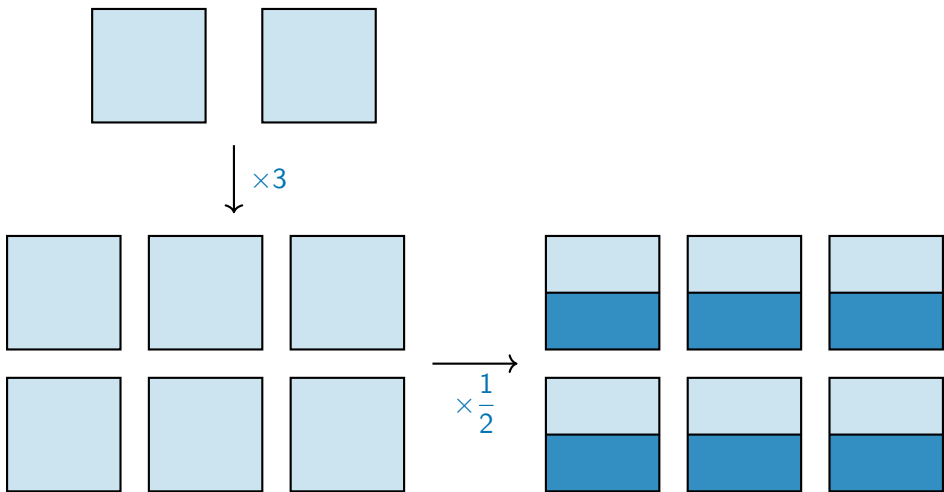
↓ $\times \frac{1}{2}$



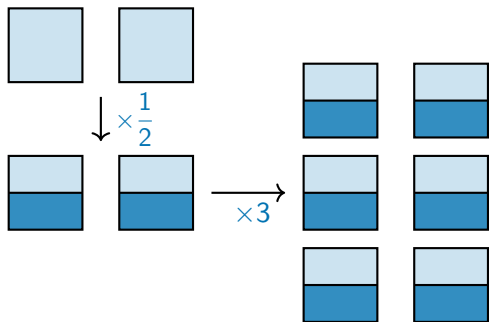
→ $\times 3$



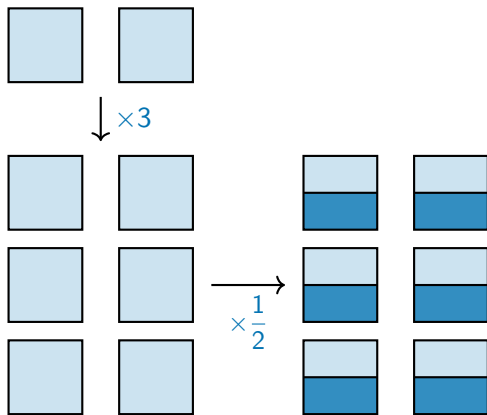
$$\frac{3}{2} \times 2 = \frac{6}{2}$$



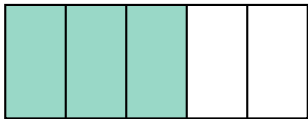
$$\frac{3}{2} \times 2 = \frac{6}{2}$$



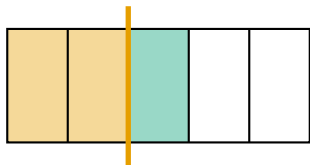
$$\frac{3}{2} \times 2 = \frac{6}{2}$$



Integer $\times$ Fraction (with an integer
division)



$$\frac{3}{5}$$

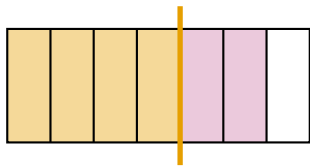
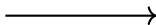


$$\frac{2}{3} \times \frac{3}{5}$$

$$\frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$$



$$\frac{6}{7}$$

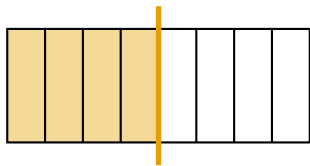


$$\frac{2}{3} \times \frac{6}{7}$$

$$\frac{2}{3} \times \frac{6}{7} = \frac{4}{7}$$

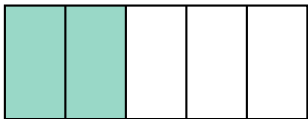


$$\frac{3}{8}$$



$$\frac{4}{3} \times \frac{3}{8}$$

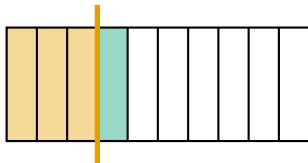
$$\frac{4}{3} \times \frac{3}{8} = \frac{4}{8}$$



$\downarrow$ $\frac{2}{5}$

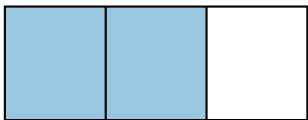


$\frac{4}{10}$

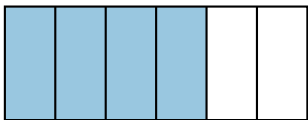


$\frac{3}{4} \times \frac{4}{10}$

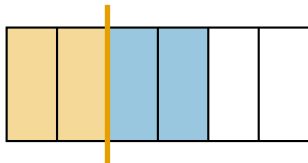
$$\frac{3}{4} \times \frac{2}{5} = \frac{3}{10}$$



$\downarrow$ $\frac{2}{3}$

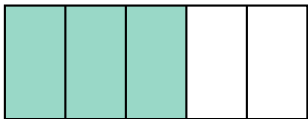


$\frac{4}{6}$

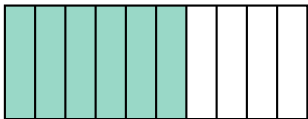


$\frac{2}{4} \times \frac{4}{6}$

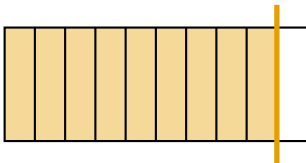
$$\frac{2}{4} \times \frac{2}{3} = \frac{2}{6}$$



↓ $\frac{3}{5}$



$\frac{6}{10}$

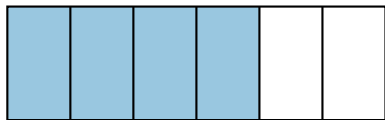


$\frac{3}{2} \times \frac{6}{10}$

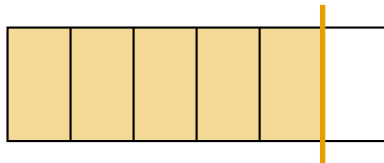
$$\frac{3}{2} \times \frac{3}{5} = \frac{9}{10}$$



$\downarrow$ $\frac{2}{3}$

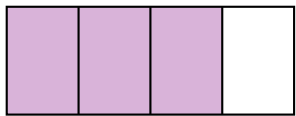


$\frac{4}{6}$



$\frac{5}{4} \times \frac{4}{6}$

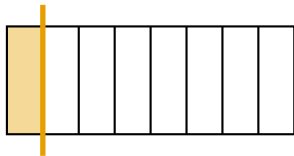
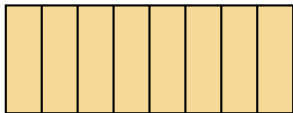
$$\frac{5}{4} \times \frac{2}{3} = \frac{5}{6}$$



↓ $\frac{3}{4}$

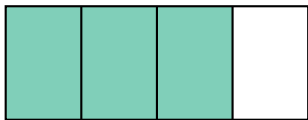


$\frac{6}{8}$



$\frac{9}{6} \times \frac{3}{4}$

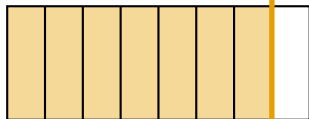
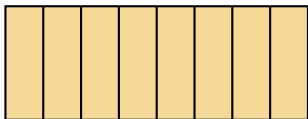
$$\frac{9}{6} \times \frac{3}{4} = \frac{9}{8}$$



$\downarrow$ $\frac{3}{4}$



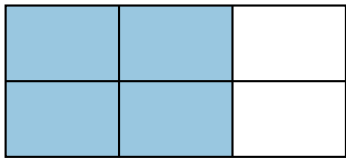
$\frac{6}{8}$



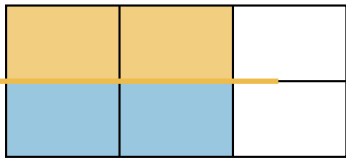
$\frac{5}{2} \times \frac{6}{8}$

$$\frac{5}{2} \times \frac{6}{8} = \frac{15}{8}$$

Alternative cracking styles

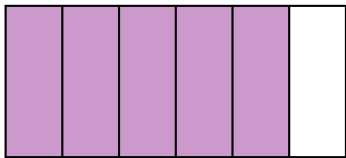


$$\frac{4}{6}$$

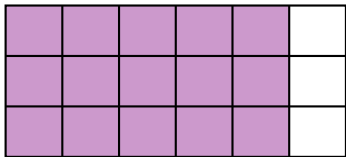


$$\frac{1}{2} \times \frac{4}{6}$$

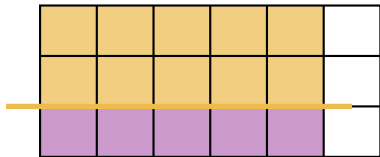
$$\frac{1}{2} \times \frac{4}{6} = \frac{2}{6}$$



↓ $\frac{5}{6}$

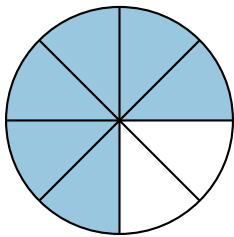


$\frac{15}{18}$

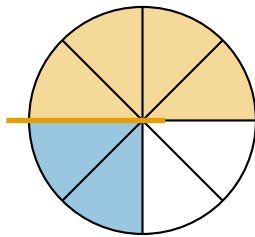


$$\frac{2}{3} \times \frac{5}{6} = \frac{10}{18}$$

$\frac{2}{3} \times \frac{15}{18}$

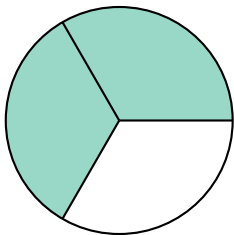


$$\frac{6}{8}$$

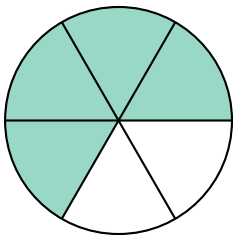


$$\frac{2}{3} \times \frac{6}{8}$$

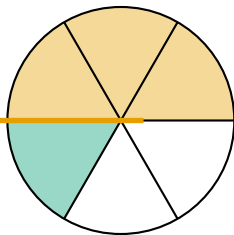
$$\frac{2}{3} \times \frac{6}{8} = \frac{3}{6}$$



$$\frac{2}{3}$$



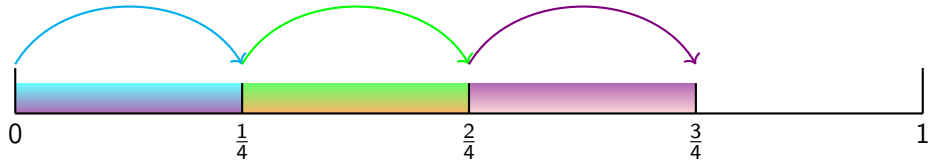
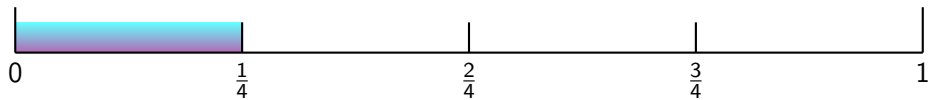
$$\frac{4}{6}$$



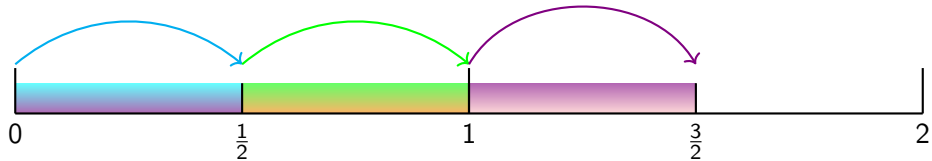
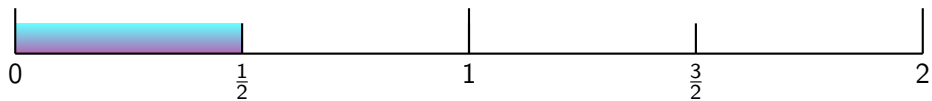
$$\frac{3}{4} \times \frac{4}{6}$$

$$\frac{3}{4} \times \frac{2}{3} = \frac{3}{6}$$

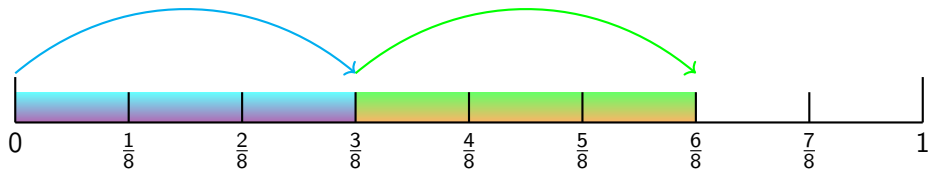
Integer $\times$ Fraction (number line)



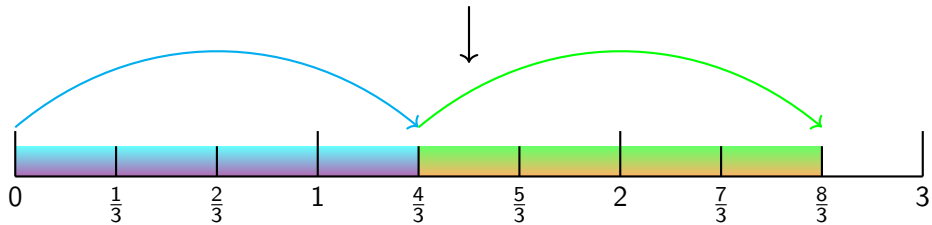
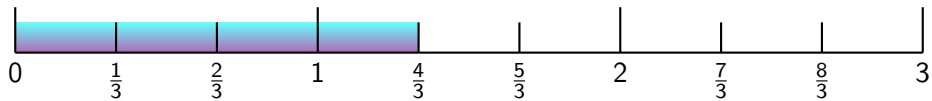
$$3 \times \frac{1}{4} = \frac{3}{4}$$



$$3 \times \frac{1}{2} = \frac{3}{2}$$

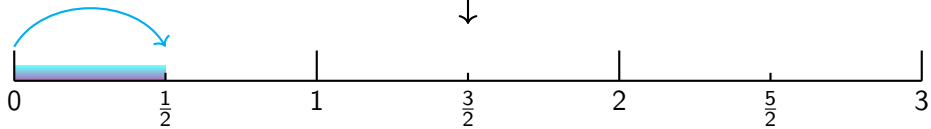
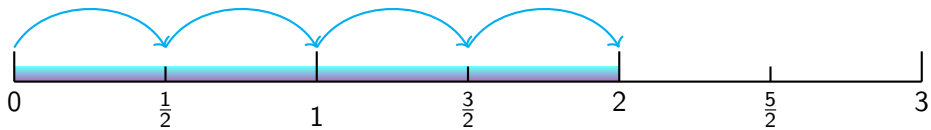


$$2 \times \frac{3}{8} = \frac{6}{8}$$

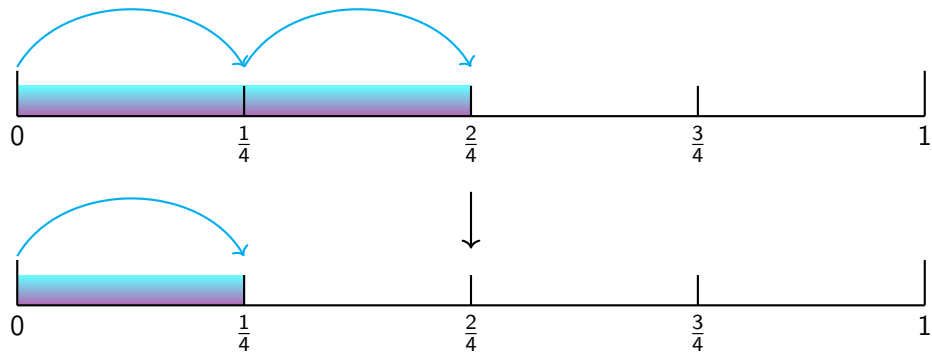


$$2 \times \frac{4}{3} = \frac{8}{3}$$

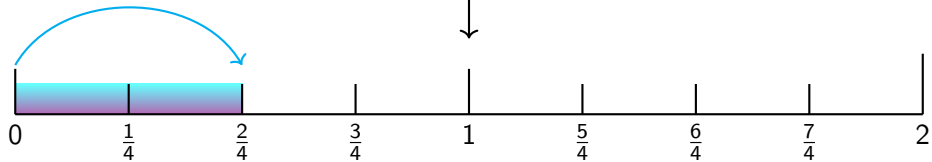
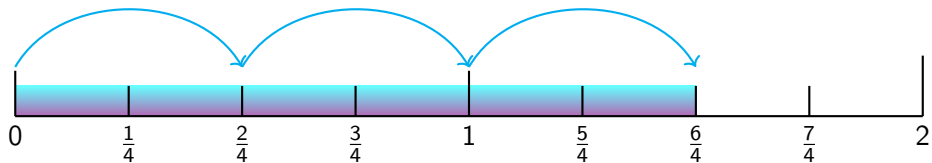
Fraction $\times$ Unit fraction (number line)



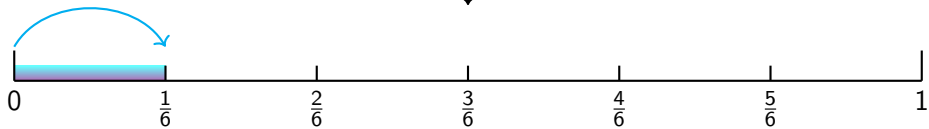
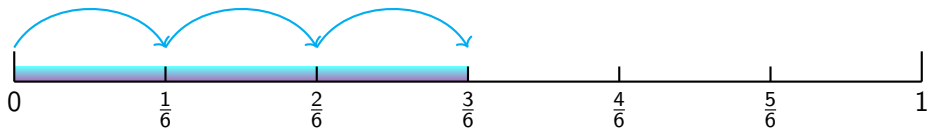
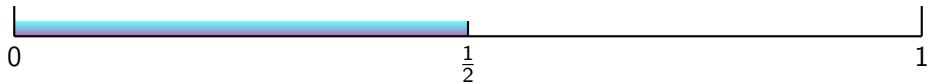
$$\frac{1}{4} \times 2 = \frac{1}{2}$$



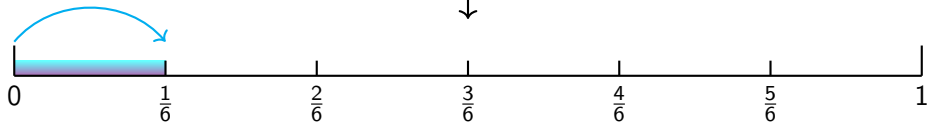
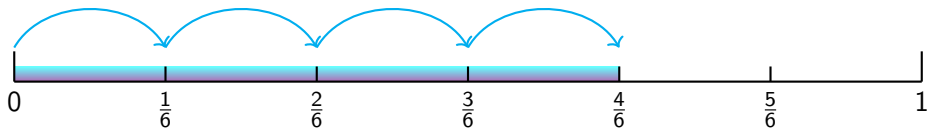
$$\frac{1}{2} \times \frac{2}{4} = \frac{1}{4}$$



$$\frac{1}{3} \times \frac{6}{4} = \frac{2}{4}$$

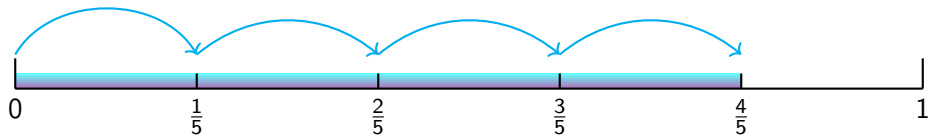
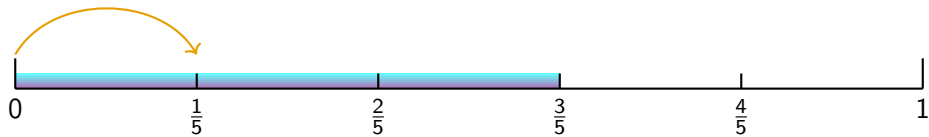


$$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

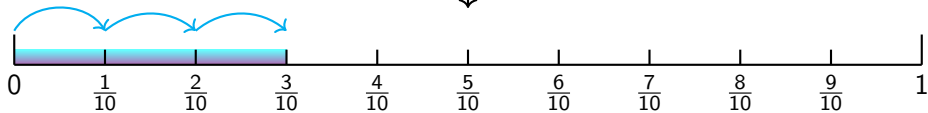
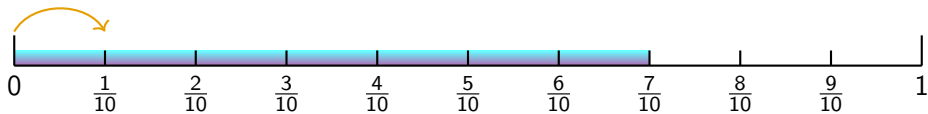
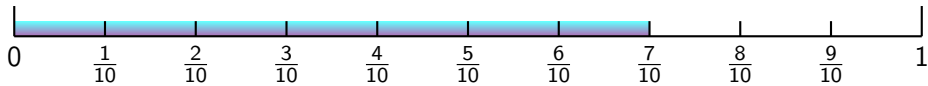


$$\frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$$

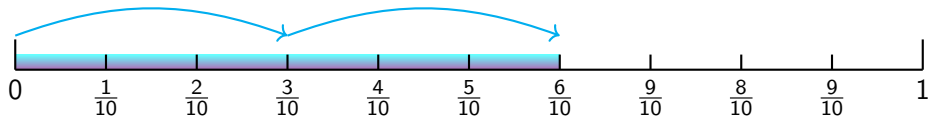
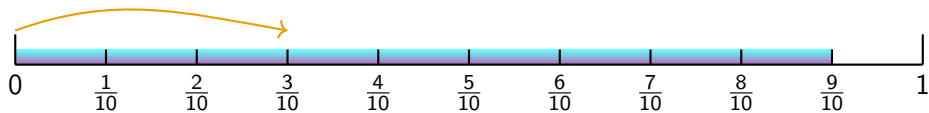
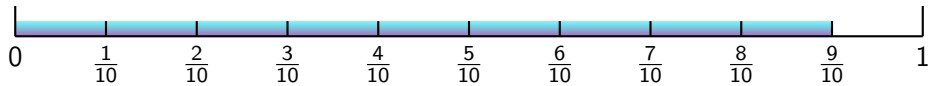
Fraction $\times$ Fraction (number line)



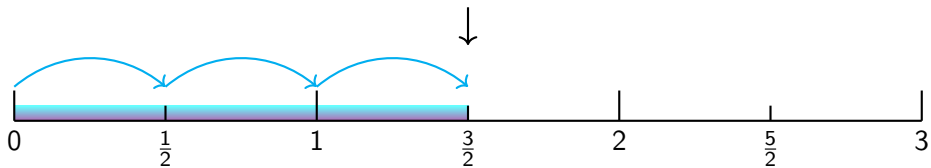
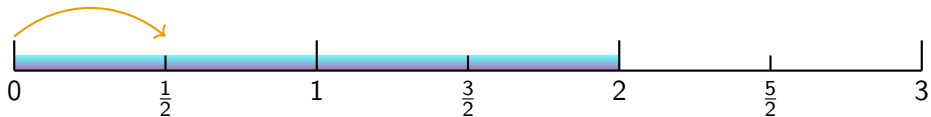
$$\frac{4}{3} \times \frac{3}{5} = \frac{4}{5}$$



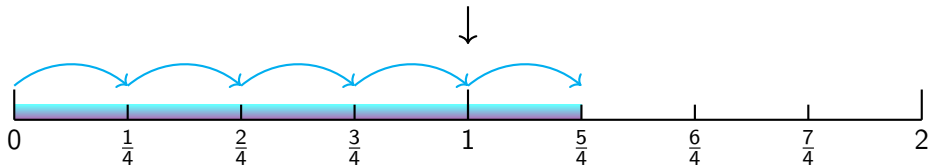
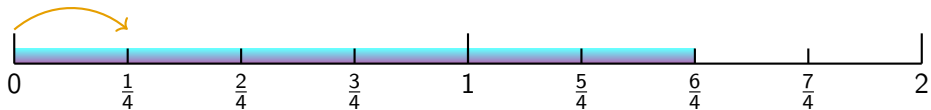
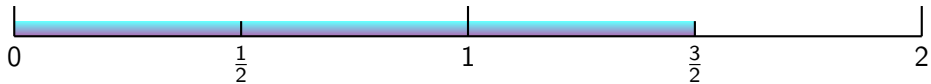
$$\frac{3}{7} \times \frac{7}{10} = \frac{3}{10}$$



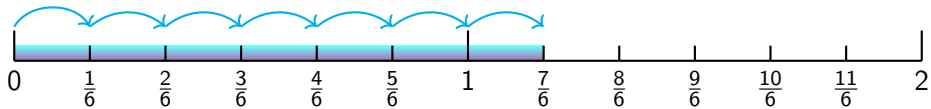
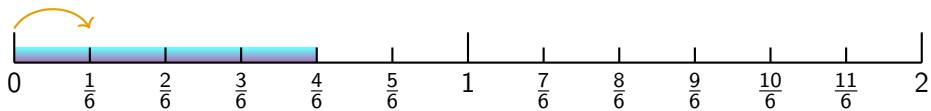
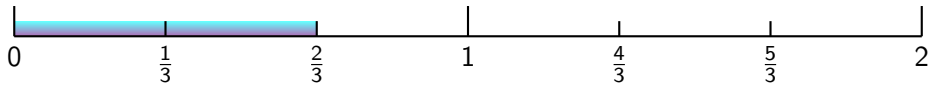
$$\frac{2}{3} \times \frac{9}{10} = \frac{6}{10}$$



$$\frac{3}{4} \times 2 = \frac{3}{2}$$

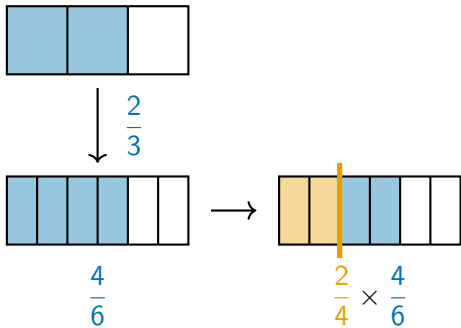


$$\frac{5}{6} \times \frac{3}{2} = \frac{5}{4}$$

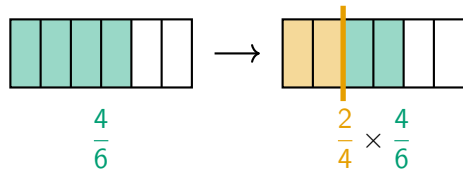


$$\frac{7}{4} \times \frac{2}{3} = \frac{7}{6}$$

Multiplication with equivalent fractions



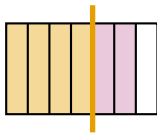
$$\frac{2}{4} \times \frac{2}{3} = \frac{2}{6}$$



$$\frac{4}{6} \times \frac{2}{3} = \frac{2}{6}$$



$$\frac{6}{7}$$

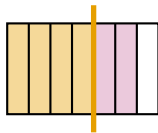


$$\frac{2}{3} \times \frac{6}{7}$$

$$\frac{2}{3} \times \frac{6}{7} = \frac{4}{7}$$



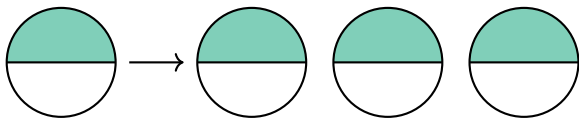
$$\frac{6}{7}$$



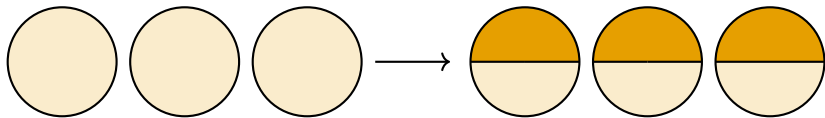
$$\frac{4}{6} \times \frac{6}{7}$$

$$\frac{4}{6} \times \frac{6}{7} = \frac{4}{7}$$

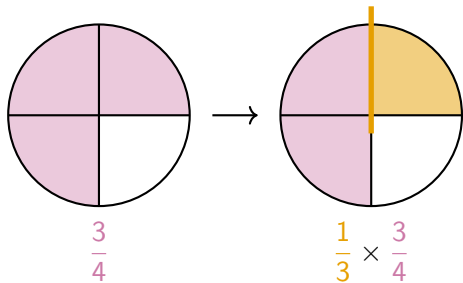
Commutativity



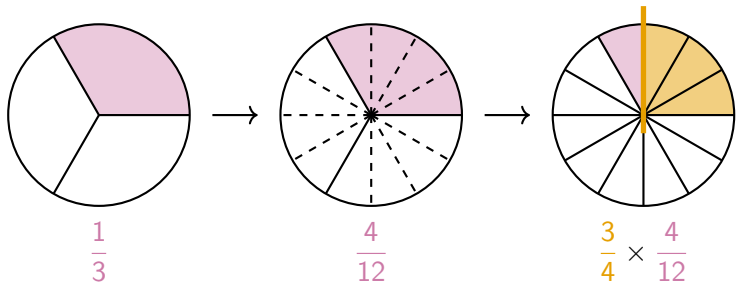
$$3 \times \frac{1}{2} = \frac{3}{2}$$



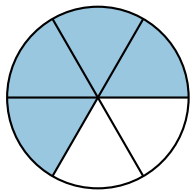
$$\frac{1}{2} \times 3 = \frac{3}{2}$$



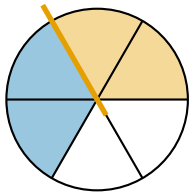
$$\frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$$



$$\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$$

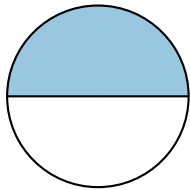


$$\frac{4}{6}$$

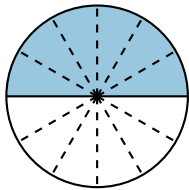


$$\frac{1}{2} \times \frac{4}{6}$$

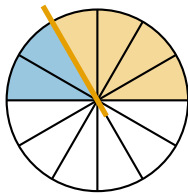
$$\frac{1}{2} \times \frac{4}{6} = \frac{1}{3}$$



$$\frac{1}{2}$$



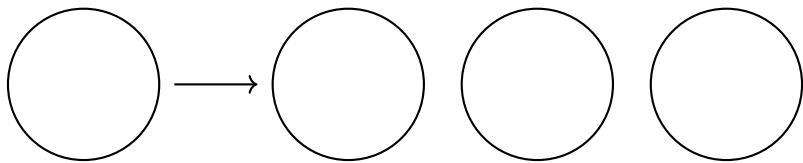
$$\frac{6}{12}$$



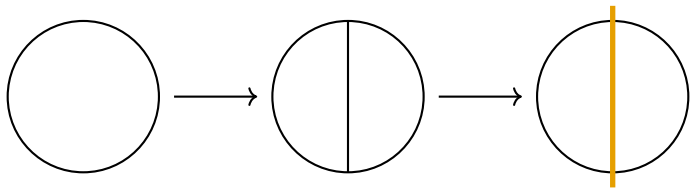
$$\frac{4}{6} \times \frac{6}{12}$$

$$\frac{4}{6} \times \frac{1}{2} = \frac{1}{3}$$

Multiplication by 0

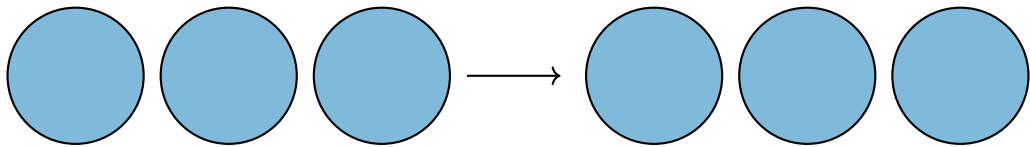


$$3 \times 0 = 0$$

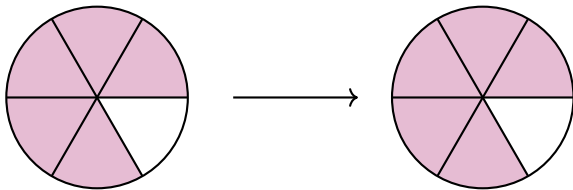


$$\frac{1}{2} \times 0 = 0$$

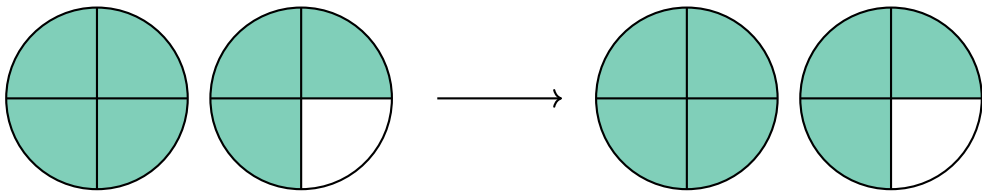
Multiplication by 1



$$1 \times 3 = 3$$

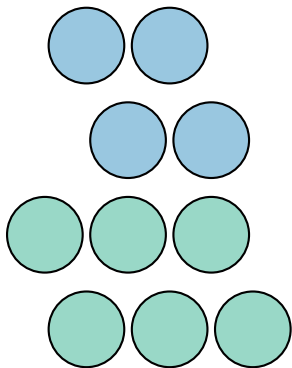
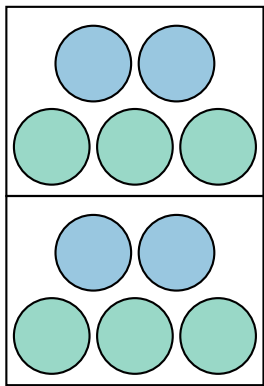


$$1 \times \frac{5}{6} = \frac{5}{6}$$

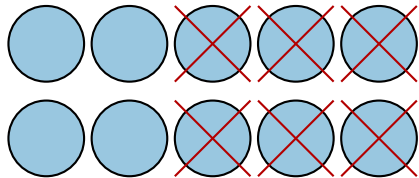
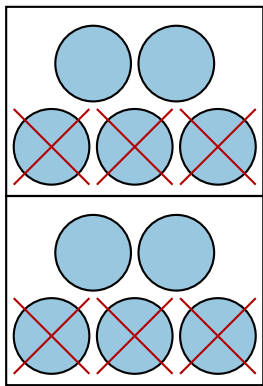


$$1 \times \frac{7}{4} = \frac{7}{4}$$

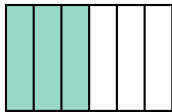
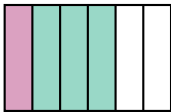
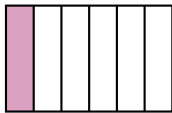
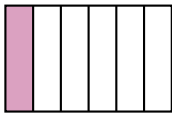
Distributivity



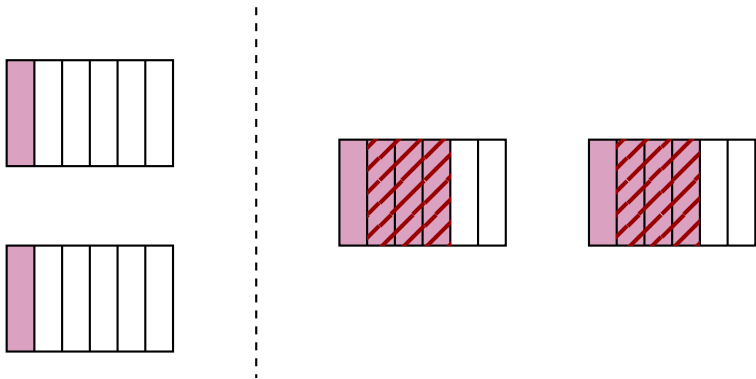
$$(2 + 3) \times 2 = 2 \times 2 + 3 \times 2$$



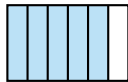
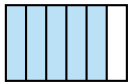
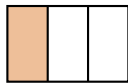
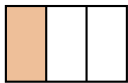
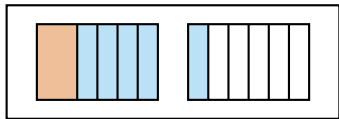
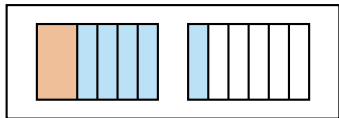
$$(5 - 3) \times 2 = 5 \times 2 - 3 \times 2$$



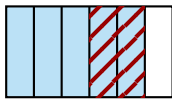
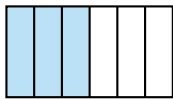
$$\left(\frac{1}{6} + \frac{3}{6} \right) \times 2 = \frac{1}{6} \times 2 + \frac{3}{6} \times 2$$



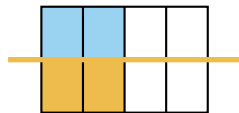
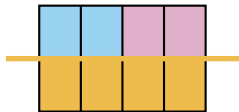
$$\left(\frac{4}{6} - \frac{3}{6}\right) \times 2 = \frac{4}{6} \times 2 - \frac{3}{6} \times 2$$



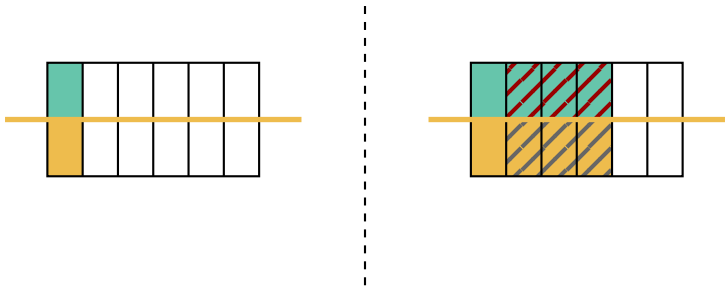
$$\left(\frac{1}{3} + \frac{5}{6} \right) \times 2 = \frac{1}{3} \times 2 + \frac{5}{6} \times 2$$



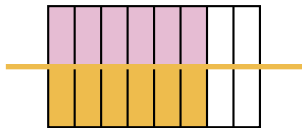
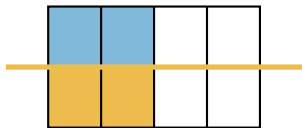
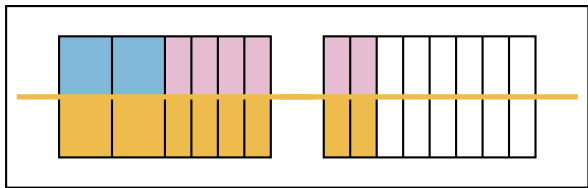
$$\left(\frac{5}{6} - \frac{1}{3}\right) \times 2 = \frac{5}{6} \times 2 - \frac{1}{3} \times 2$$



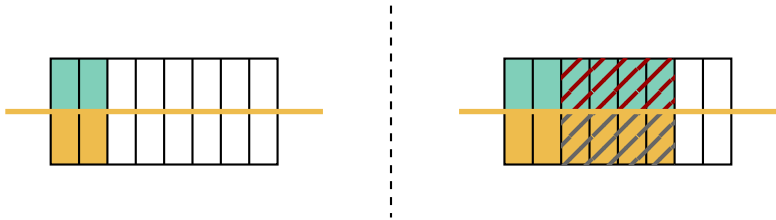
$$\left(\frac{2}{4} + \frac{2}{4} \right) \times \frac{1}{2} = \frac{2}{4} \times \frac{1}{2} + \frac{2}{4} \times \frac{1}{2}$$



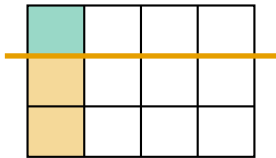
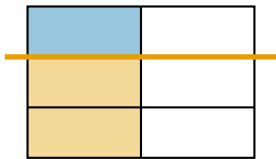
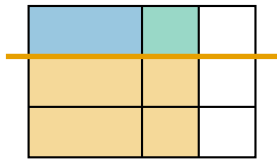
$$\left(\frac{4}{6} - \frac{3}{6}\right) \times \frac{1}{2} = \frac{4}{6} \times \frac{1}{2} - \frac{3}{6} \times \frac{1}{2}$$



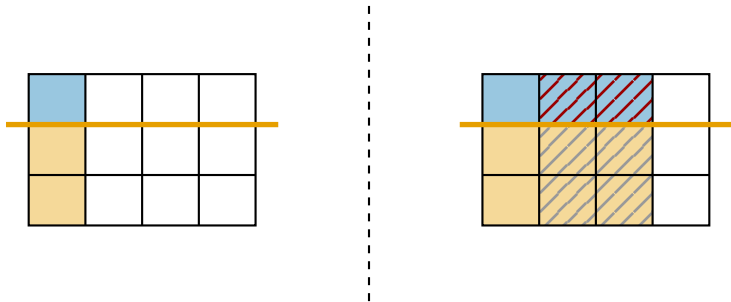
$$\left(\frac{2}{4} + \frac{6}{8}\right) \times \frac{1}{2} = \frac{2}{4} \times \frac{1}{2} + \frac{6}{8} \times \frac{1}{2}$$



$$\left(\frac{6}{8} - \frac{2}{4} \right) \times \frac{1}{2} = \frac{6}{8} \times \frac{1}{2} - \frac{2}{4} \times \frac{1}{2}$$

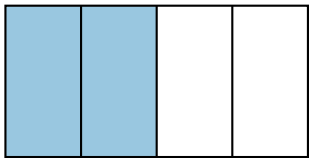


$$\left(\frac{1}{2} + \frac{1}{4}\right) \times \frac{2}{3} = \frac{1}{2} \times \frac{2}{3} + \frac{1}{4} \times \frac{2}{3}$$

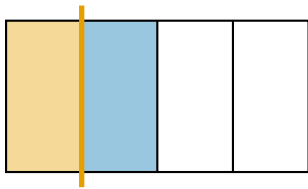
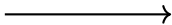


$$\left(\frac{3}{4} - \frac{1}{2} \right) \times \frac{2}{3} = \frac{3}{4} \times \frac{2}{3} - \frac{1}{2} \times \frac{2}{3}$$

On the result (times a proper fraction)

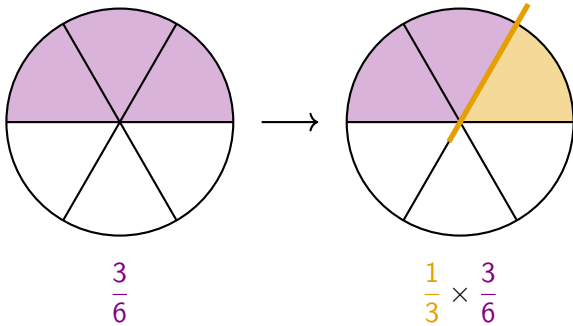


$$\frac{2}{4}$$

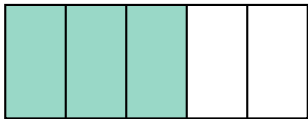


$$\frac{1}{2} \times \frac{2}{4}$$

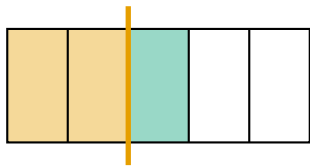
$$\frac{2}{4} > \frac{1}{2} \times \frac{2}{4}$$



$$\frac{3}{6} > \frac{1}{3} \times \frac{3}{6}$$



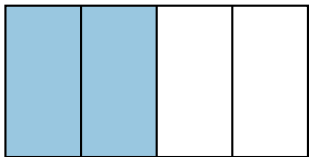
$$\frac{3}{5}$$



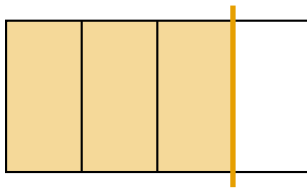
$$\frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$$

$$\frac{3}{5} > \frac{2}{3} \times \frac{3}{5}$$

On the result (times an improper fraction)

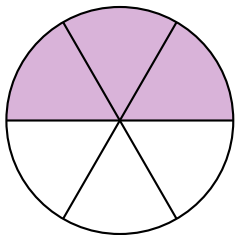


$$\frac{2}{4}$$

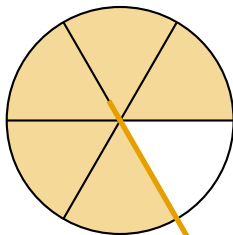


$$\frac{3}{2} \times \frac{2}{4}$$

$$\frac{2}{4} < \frac{3}{2} \times \frac{2}{4}$$

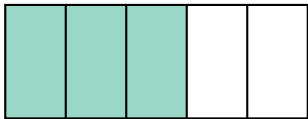


$$\frac{3}{6}$$

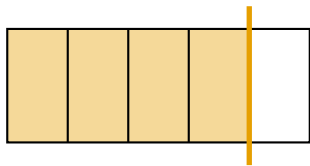


$$\frac{5}{3} \times \frac{3}{6}$$

$$\frac{3}{6} < \frac{5}{3} \times \frac{3}{6}$$



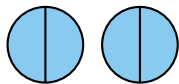
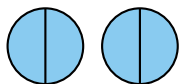
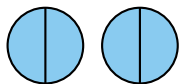
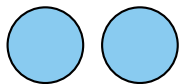
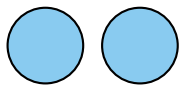
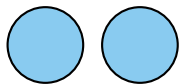
$$\frac{3}{5}$$



$$\frac{4}{3} \times \frac{3}{5}$$

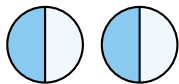
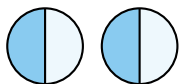
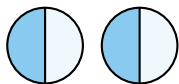
$$\frac{3}{5} < \frac{4}{3} \times \frac{3}{5}$$

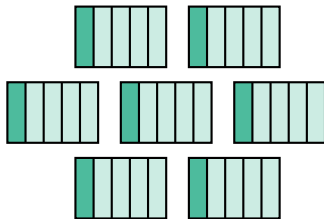
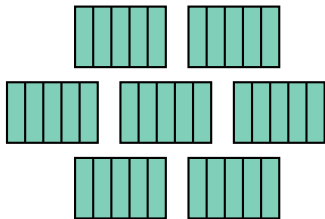
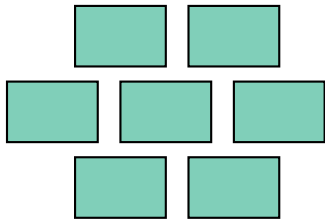
Dividing integers



$$6 : 2 = 3$$

$$6 \times \frac{1}{2} = 3$$





$$7 : 5 = \frac{7}{5}$$

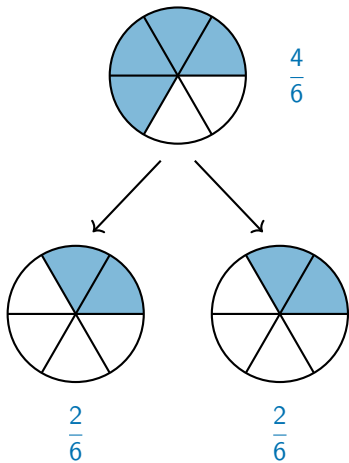
$$7 \times \frac{1}{5} = \frac{7}{5}$$



$$3 : 5 = \frac{3}{5}$$

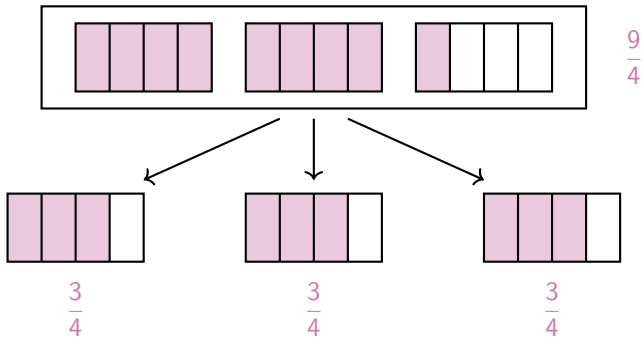
$$3 \times \frac{1}{5} = \frac{3}{5}$$

Dividing a fraction by an integer (with
a partition)



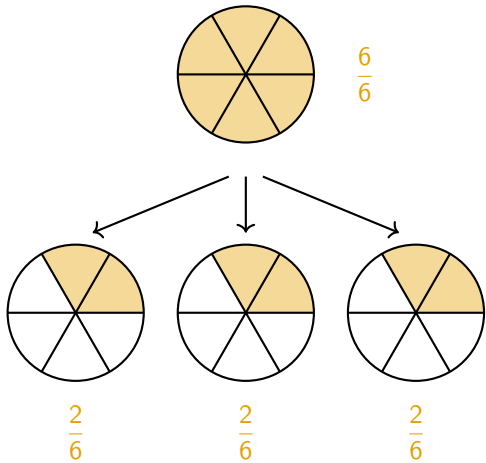
$$\frac{4}{6} : 2 = \frac{2}{6}$$

$$\frac{4}{6} \times \frac{1}{2} = \frac{2}{6}$$



$$\frac{9}{4} : 3 = \frac{3}{4}$$

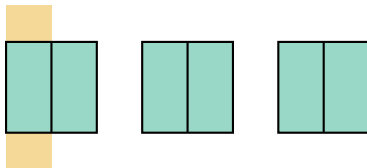
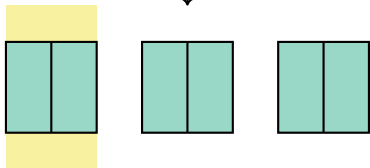
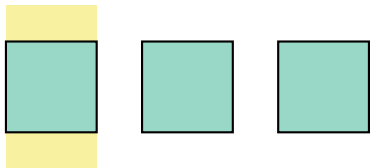
$$\frac{9}{4} \times \frac{1}{3} = \frac{3}{4}$$



$$\frac{6}{6} : 3 = \frac{2}{6}$$

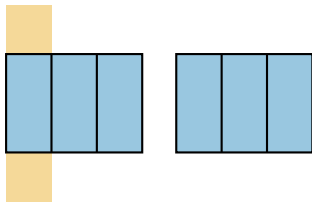
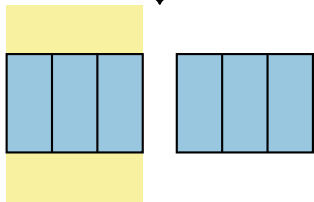
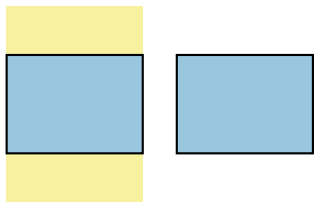
$$\frac{6}{6} \times \frac{1}{3} = \frac{2}{6}$$

Dividing an integer by a unit fraction



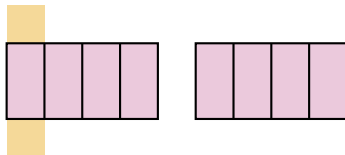
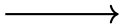
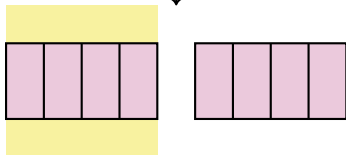
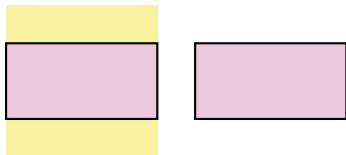
$$3 : \frac{1}{2} = 6$$

$$3 \times 2 = 6$$



$$2 : \frac{1}{3} = 6$$

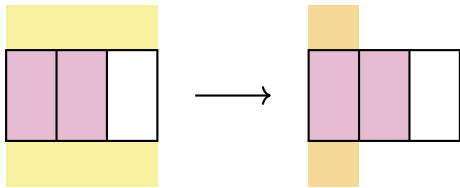
$$2 \times 3 = 6$$



$$2 : \frac{1}{4} = 8$$

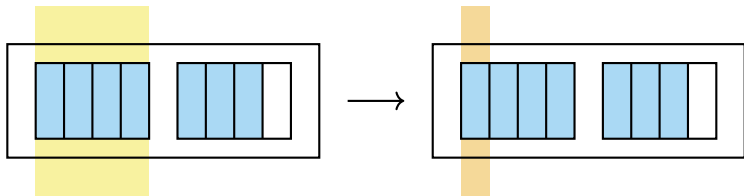
$$2 \times 4 = 8$$

Dividing a fraction by a unit fraction



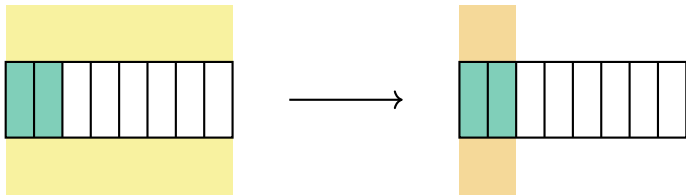
$$\frac{2}{3} : \frac{1}{3} = 2$$

$$\frac{2}{3} \times 3 = 2$$



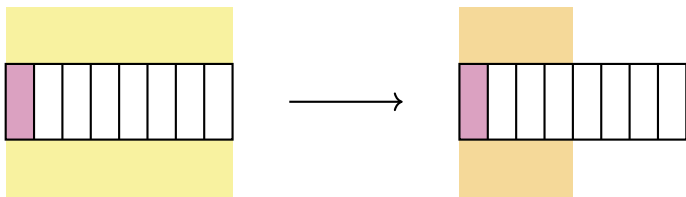
$$\frac{7}{4} : \frac{1}{4} = 7$$

$$\frac{7}{4} \times 4 = 7$$



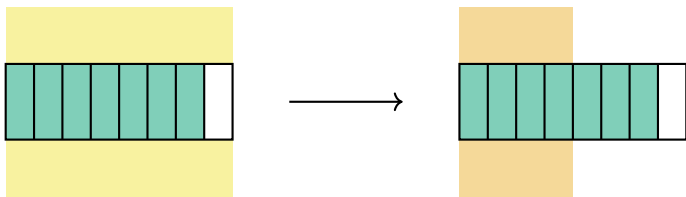
$$\frac{2}{8} : \frac{1}{4} = 1$$

$$\frac{2}{8} \times 4 = 1$$



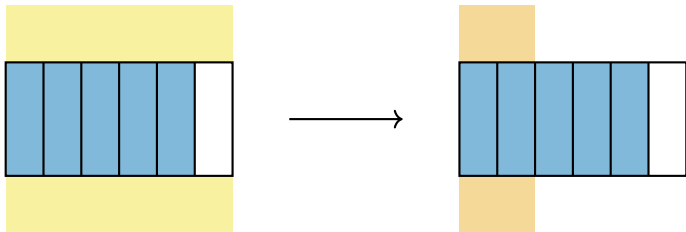
$$\frac{1}{8} \div \frac{1}{2} = \frac{1}{4}$$

$$\frac{1}{8} \times 2 = \frac{1}{4}$$



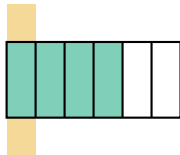
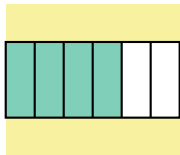
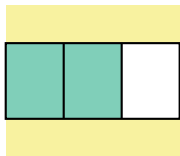
$$\frac{7}{8} \div \frac{1}{2} = \frac{7}{4}$$

$$\frac{7}{8} \times 2 = \frac{7}{4}$$



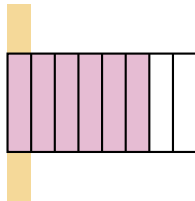
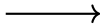
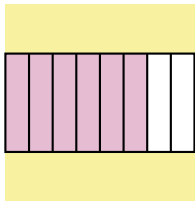
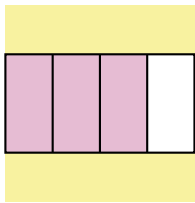
$$\frac{5}{6} \div \frac{1}{3} = \frac{5}{2}$$

$$\frac{5}{6} \times 3 = \frac{5}{2}$$



$$\frac{2}{3} : \frac{1}{6} = 4$$

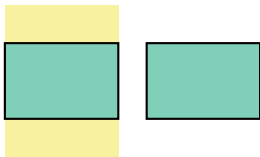
$$\frac{2}{3} \times 6 = 4$$



$$\frac{3}{4} : \frac{1}{8} = 6$$

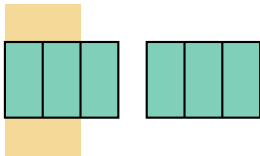
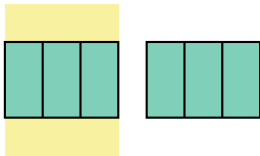
$$\frac{3}{4} \times 8 = 6$$

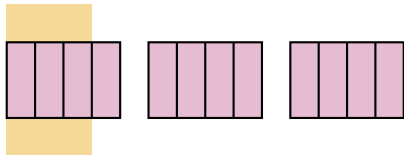
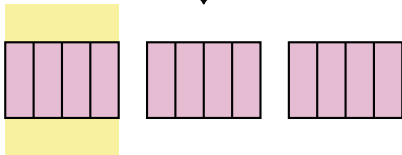
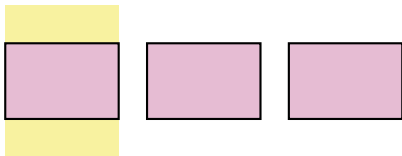
Dividing an integer by a fraction



$$2 : \frac{2}{3} = 3$$

$$2 \times \frac{3}{2} = 3$$

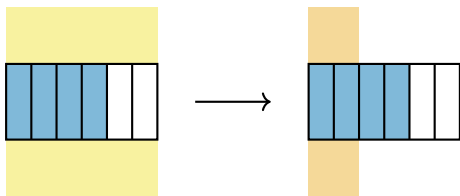




$$3 : \frac{3}{4} = 4$$

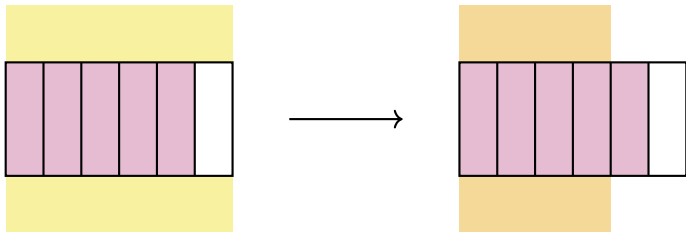
$$3 \times \frac{4}{3} = 4$$

Dividing a fraction by a proper fraction



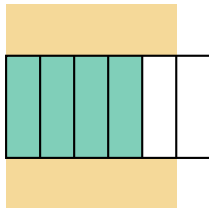
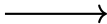
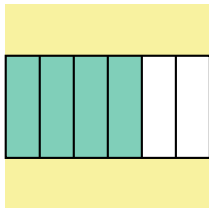
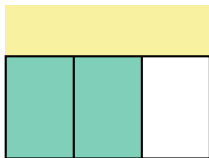
$$\frac{4}{6} : \frac{2}{6} = 2$$

$$\frac{4}{6} \times \frac{6}{2} = 2$$



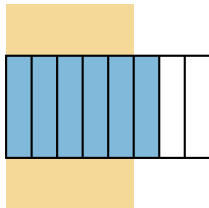
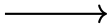
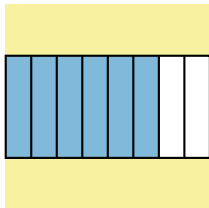
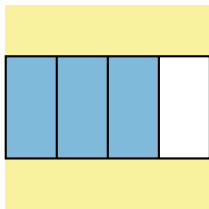
$$\frac{5}{6} \div \frac{2}{3} = \frac{5}{4}$$

$$\frac{5}{6} \times \frac{3}{2} = \frac{5}{4}$$



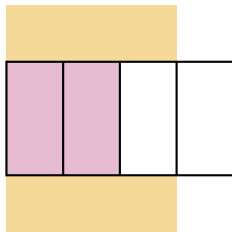
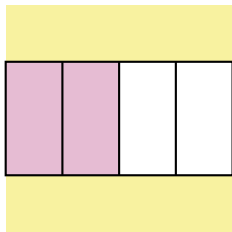
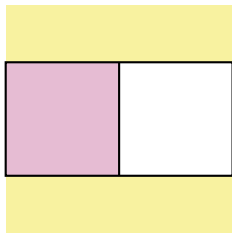
$$\frac{2}{3} : \frac{5}{6} = \frac{4}{5}$$

$$\frac{2}{3} \times \frac{6}{5} = \frac{4}{5}$$



$$\frac{3}{4} : \frac{5}{8} = \frac{6}{5}$$

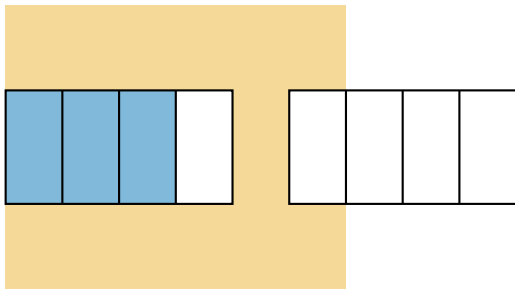
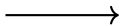
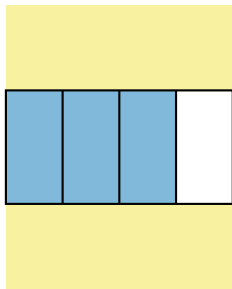
$$\frac{3}{4} \times \frac{8}{5} = \frac{6}{5}$$



$$\frac{1}{2} : \frac{3}{4} = \frac{2}{3}$$

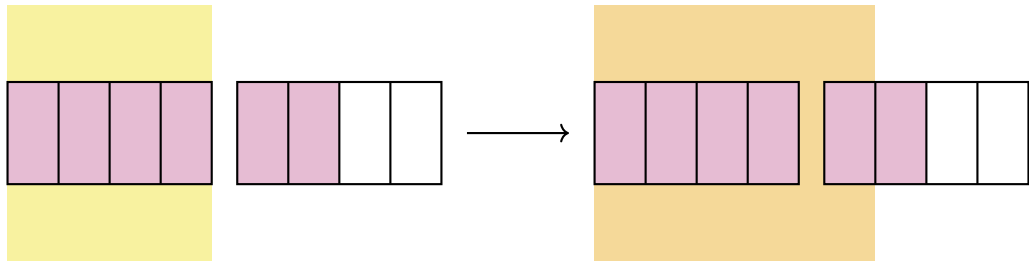
$$\frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$$

Dividing a fraction by an improper fraction



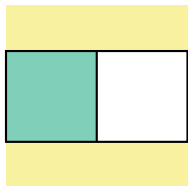
$$\frac{3}{4} : \frac{5}{4} = \frac{3}{5}$$

$$\frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$$



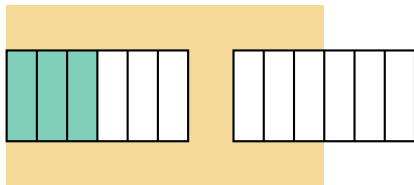
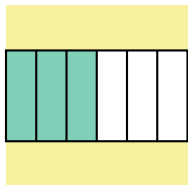
$$\frac{6}{4} \div \frac{5}{4} = \frac{6}{5}$$

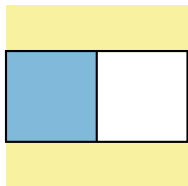
$$\frac{6}{4} \times \frac{4}{5} = \frac{6}{5}$$



$$\frac{1}{2} \div \frac{9}{6} = \frac{3}{9}$$

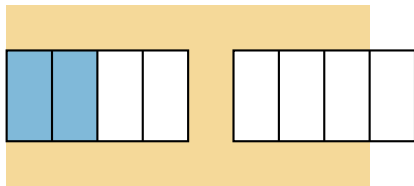
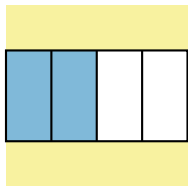
$$\frac{1}{2} \times \frac{6}{9} = \frac{3}{9}$$





$$\frac{1}{2} \div \frac{7}{4} = \frac{2}{7}$$

$$\frac{1}{2} \times \frac{4}{7} = \frac{2}{7}$$

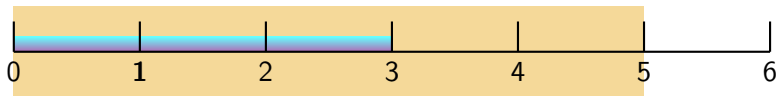
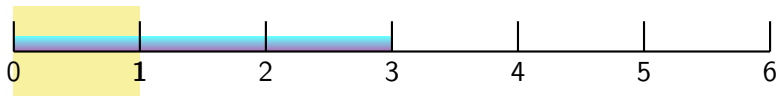


Dividing integers (number line)



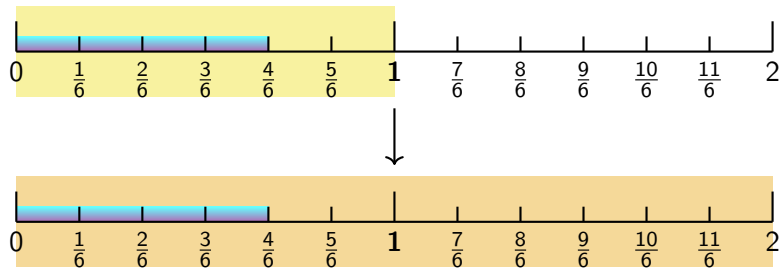
$$6 : 2 = 3$$

$$6 \times \frac{1}{2} = 3$$



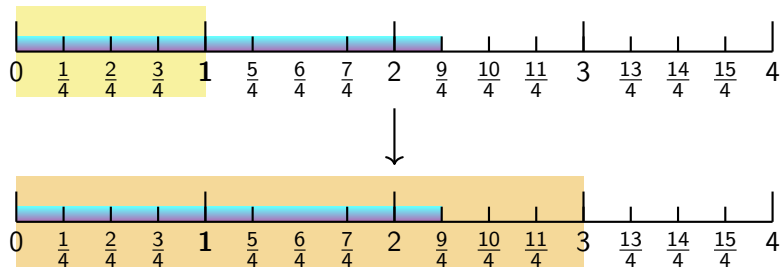
$$3 : 5 = \frac{3}{5}$$

$$3 \times \frac{1}{5} = \frac{3}{5}$$



$$\frac{4}{6} : 2 = \frac{4}{12}$$

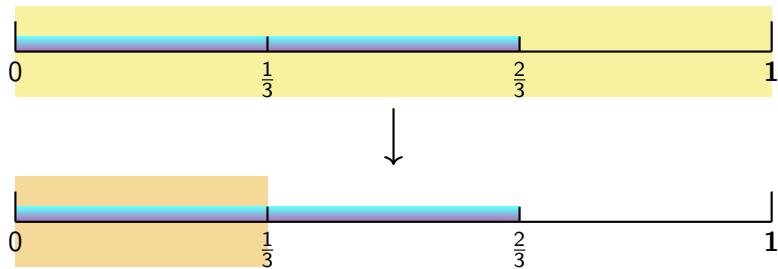
$$\frac{4}{6} \times \frac{1}{2} = \frac{4}{12}$$



$$\frac{9}{4} : 3 = \frac{9}{12}$$

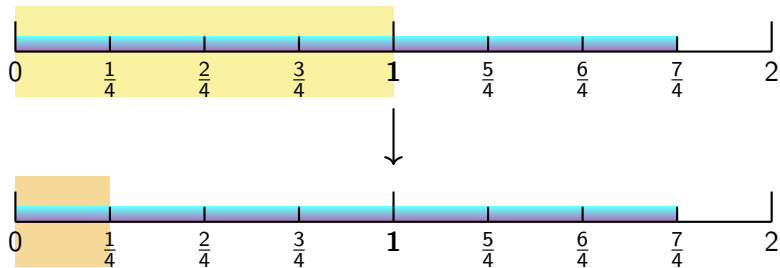
$$\frac{9}{4} \times \frac{1}{3} = \frac{9}{12}$$

Dividing by a unit fraction (number line)



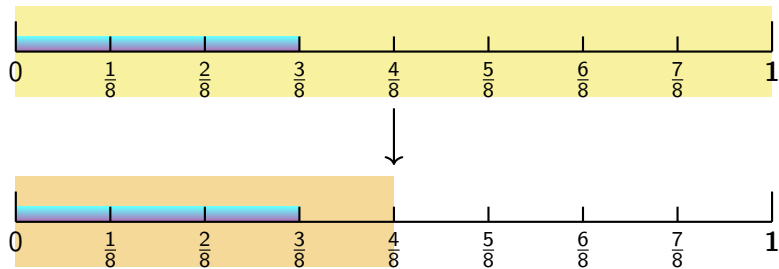
$$\frac{2}{3} : \frac{1}{3} = 2$$

$$\frac{2}{3} \times 3 = 2$$



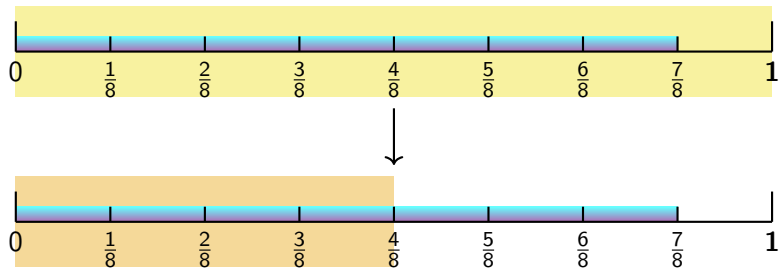
$$\frac{7}{4} : \frac{1}{4} = 7$$

$$\frac{7}{4} \times 4 = 7$$



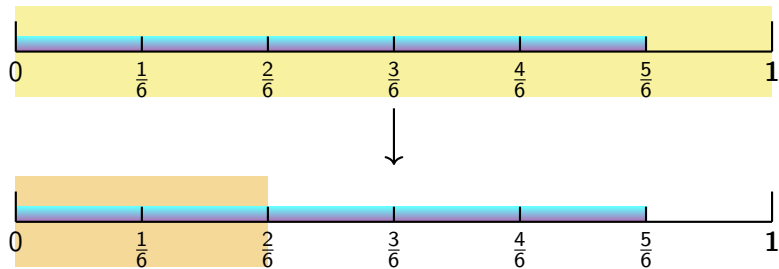
$$\frac{3}{8} : \frac{1}{2} = \frac{3}{4}$$

$$\frac{3}{8} \times 2 = \frac{3}{4}$$



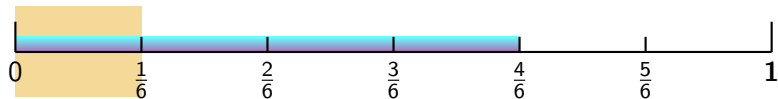
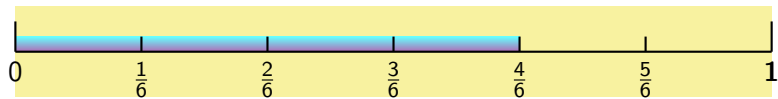
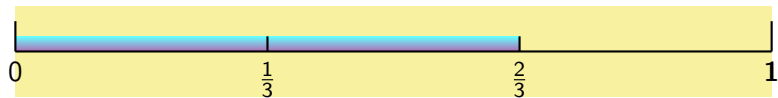
$$\frac{7}{8} : \frac{1}{2} = \frac{7}{4}$$

$$\frac{7}{8} \times 2 = \frac{7}{4}$$



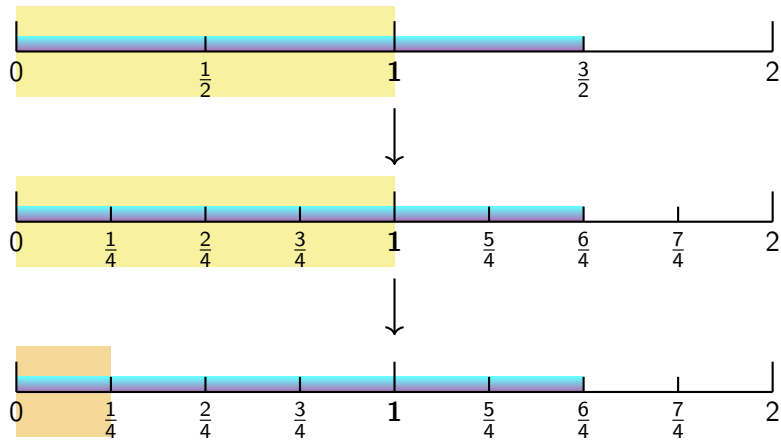
$$\frac{5}{6} : \frac{1}{3} = \frac{5}{2}$$

$$\frac{5}{6} \times 3 = \frac{5}{2}$$



$$\frac{2}{3} : \frac{1}{6} = 4$$

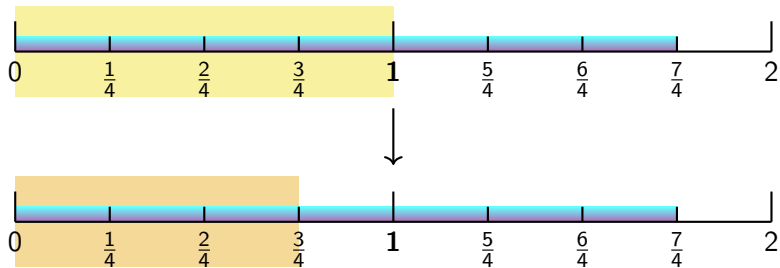
$$\frac{2}{3} \times 6 = 4$$



$$\frac{3}{2} : \frac{1}{4} = 6$$

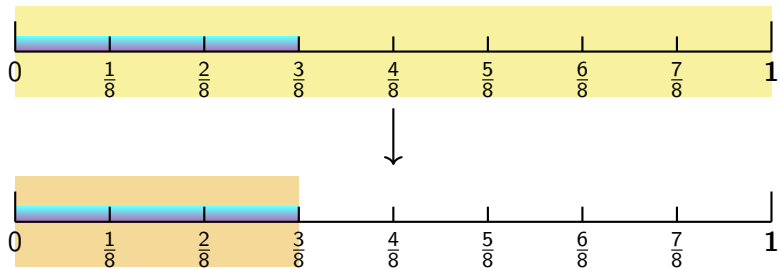
$$\frac{3}{2} \times 4 = 6$$

Dividing by a fraction (number line)



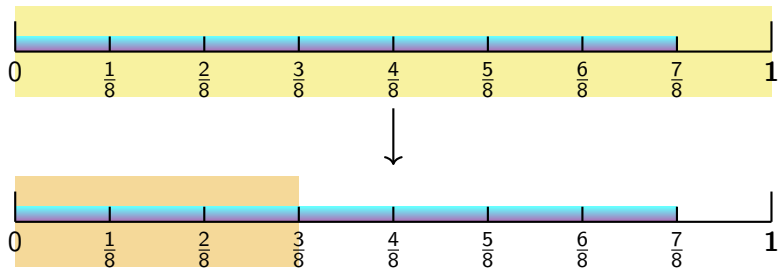
$$\frac{7}{4} : \frac{3}{4} = \frac{7}{3}$$

$$\frac{7}{4} \times \frac{4}{3} = \frac{7}{3}$$



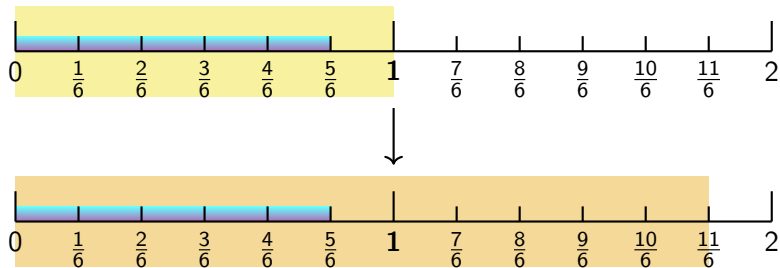
$$\frac{3}{8} : \frac{3}{8} = \frac{3}{3}$$

$$\frac{3}{8} \times \frac{8}{3} = \frac{3}{3}$$



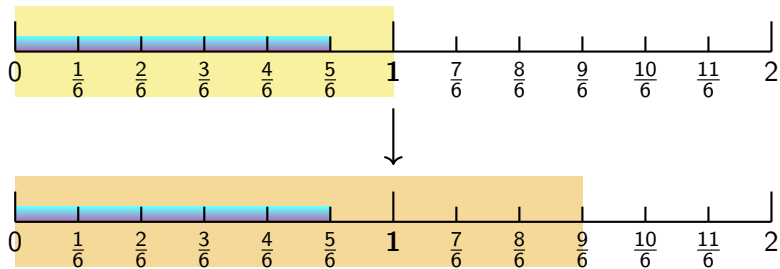
$$\frac{7}{8} : \frac{3}{8} = \frac{7}{3}$$

$$\frac{7}{8} \times \frac{8}{3} = \frac{7}{3}$$



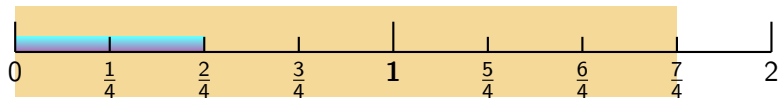
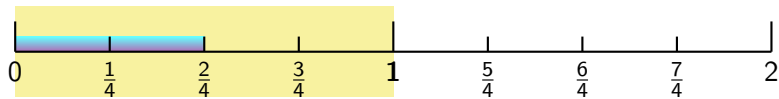
$$\frac{5}{6} : \frac{11}{6} = \frac{5}{11}$$

$$\frac{5}{6} \times \frac{6}{11} = \frac{5}{11}$$



$$\frac{5}{6} : \frac{3}{2} = \frac{5}{9}$$

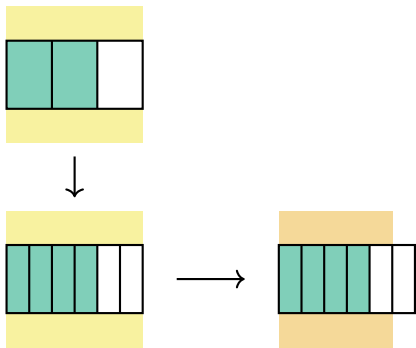
$$\frac{5}{6} \times \frac{2}{3} = \frac{5}{9}$$



$$\frac{1}{2} : \frac{7}{4} = \frac{2}{7}$$

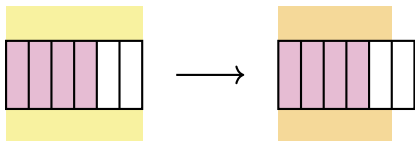
$$\frac{1}{2} \times \frac{4}{7} = \frac{2}{7}$$

Division with equivalent fractions



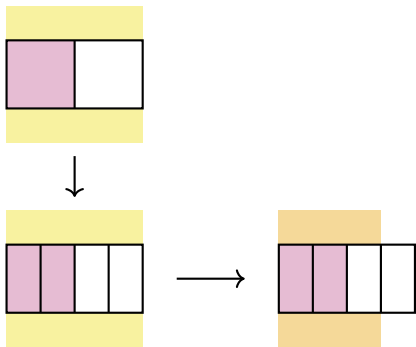
$$\frac{2}{3} \div \frac{5}{6} = \frac{4}{5}$$

$$\frac{2}{3} \times \frac{6}{5} = \frac{4}{5}$$



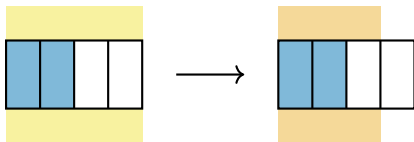
$$\frac{4}{6} \div \frac{5}{6} = \frac{4}{5}$$

$$\frac{4}{6} \times \frac{6}{5} = \frac{4}{5}$$



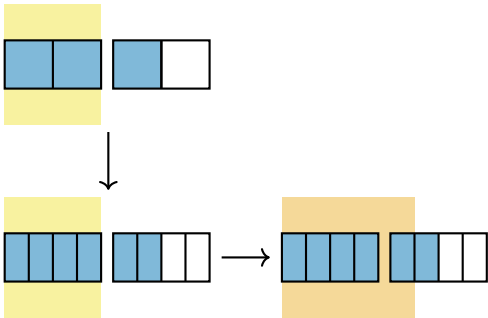
$$\frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$$

$$\frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$$



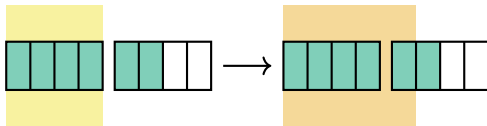
$$\frac{2}{4} \div \frac{3}{4} = \frac{2}{3}$$

$$\frac{2}{4} \times \frac{4}{3} = \frac{2}{3}$$



$$\frac{3}{2} : \frac{5}{4} = \frac{6}{5}$$

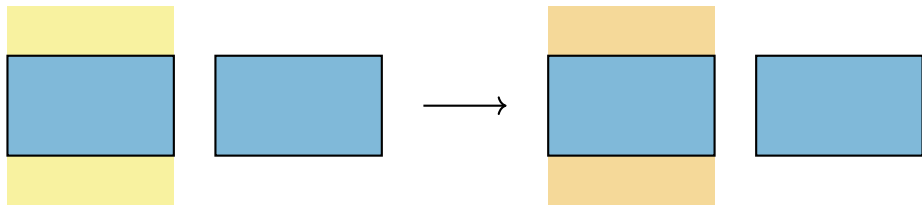
$$\frac{3}{2} \times \frac{4}{5} = \frac{6}{5}$$



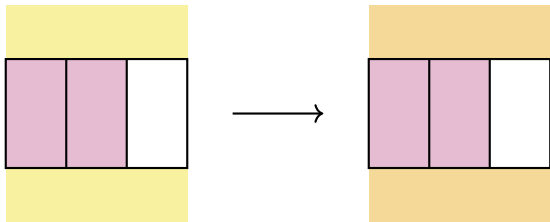
$$\frac{6}{4} : \frac{5}{4} = \frac{6}{5}$$

$$\frac{6}{4} \times \frac{4}{5} = \frac{6}{5}$$

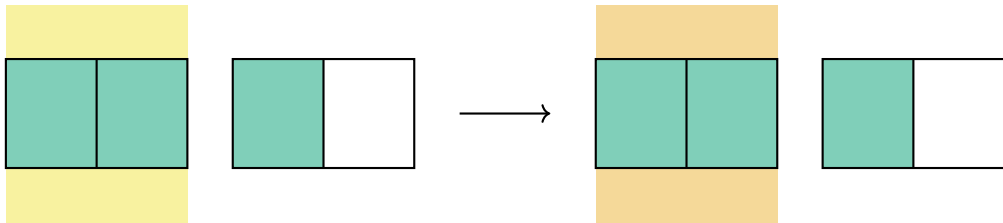
Division by 1



$$2 : 1 = 2$$

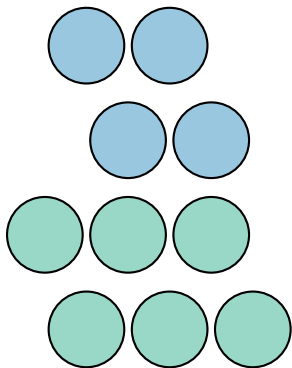
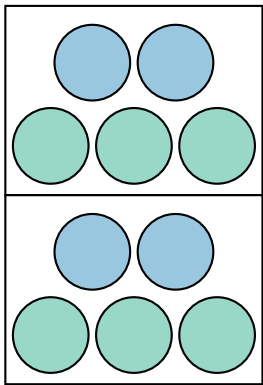


$$\frac{2}{3} : 1 = \frac{2}{3}$$



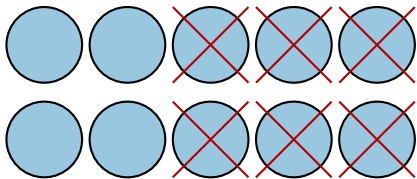
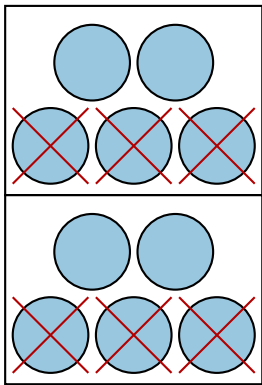
$$\frac{3}{2} : 1 = \frac{3}{2}$$

Distributivity



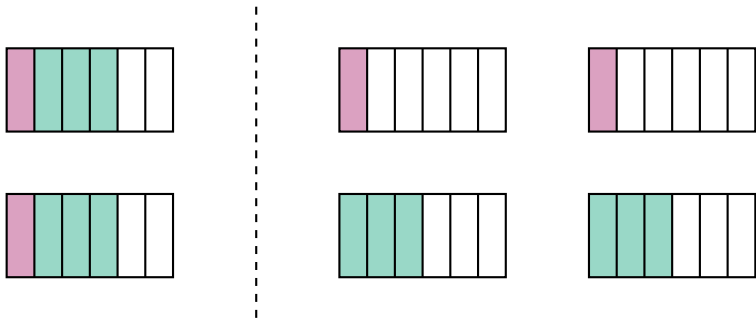
$$(2 + 3) : \frac{1}{2} = 2 : \frac{1}{2} + 3 : \frac{1}{2}$$

$$(2 + 3) \times 2 = 2 \times 2 + 3 \times 2$$



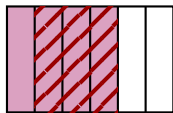
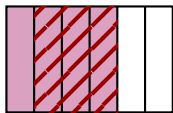
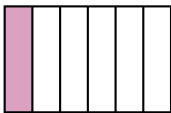
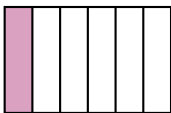
$$(5 - 3) : \frac{1}{2} = 5 : \frac{1}{2} - 3 : \frac{1}{2}$$

$$(5 - 3) \times 2 = 5 \times 2 - 3 \times 2$$



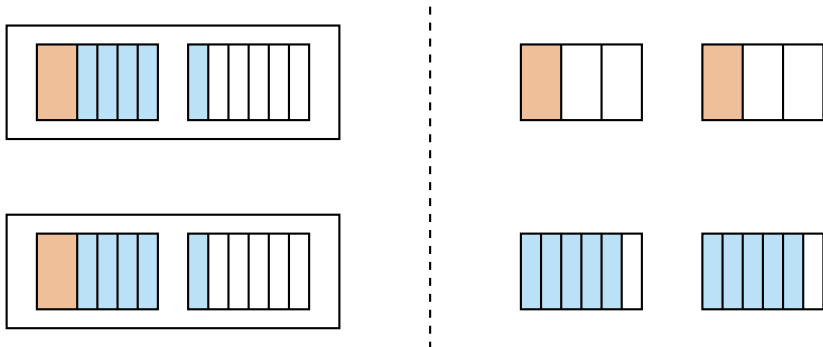
$$\left(\frac{1}{6} + \frac{3}{6}\right) : \frac{1}{2} = \frac{1}{6} : \frac{1}{2} + \frac{3}{6} : \frac{1}{2}$$

$$\left(\frac{1}{6} + \frac{3}{6}\right) \times 2 = \frac{1}{6} \times 2 + \frac{3}{6} \times 2$$



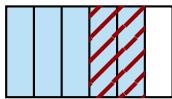
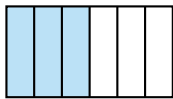
$$\left(\frac{4}{6} - \frac{3}{6}\right) : \frac{1}{2} = \frac{4}{6} : \frac{1}{2} - \frac{3}{6} : \frac{1}{2}$$

$$\left(\frac{4}{6} - \frac{3}{6}\right) \times 2 = \frac{4}{6} \times 2 - \frac{3}{6} \times 2$$



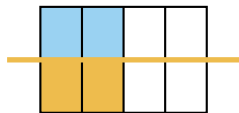
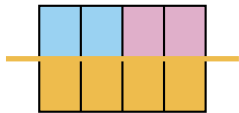
$$\left(\frac{1}{3} + \frac{5}{6}\right) : \frac{1}{2} = \frac{1}{3} : \frac{1}{2} + \frac{5}{6} : \frac{1}{2}$$

$$\left(\frac{1}{3} + \frac{5}{6}\right) \times 2 = \frac{1}{3} \times 2 + \frac{5}{6} \times 2$$



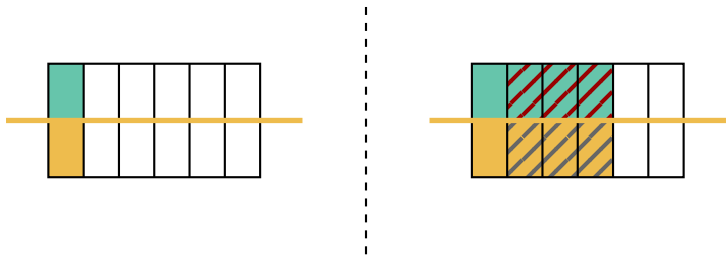
$$\left(\frac{5}{6} - \frac{1}{3}\right) \div \frac{1}{2} = \frac{5}{6} \div \frac{1}{2} - \frac{1}{3} \div \frac{1}{2}$$

$$\left(\frac{5}{6} - \frac{1}{3}\right) \times 2 = \frac{5}{6} \times 2 - \frac{1}{3} \times 2$$



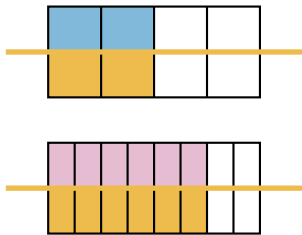
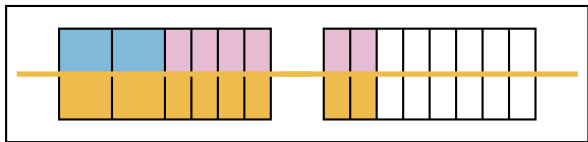
$$\left(\frac{2}{4} + \frac{2}{4}\right) : 2 = \frac{2}{4} : 2 + \frac{2}{4} : 2$$

$$\left(\frac{2}{4} + \frac{2}{4}\right) \times \frac{1}{2} = \frac{2}{4} \times \frac{1}{2} + \frac{2}{4} \times \frac{1}{2}$$



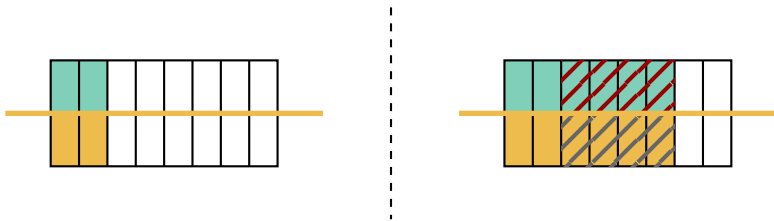
$$\left(\frac{4}{6} - \frac{2}{6}\right) : 2 = \frac{4}{6} : 2 - \frac{2}{6} : 2$$

$$\left(\frac{4}{6} - \frac{2}{6}\right) \times \frac{1}{2} = \frac{4}{6} \times \frac{1}{2} - \frac{2}{6} \times \frac{1}{2}$$



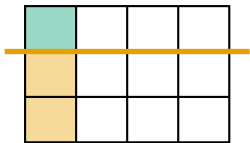
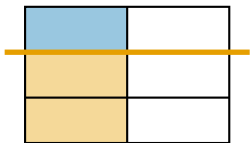
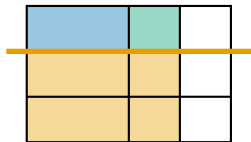
$$\left(\frac{2}{4} + \frac{6}{8}\right) : 2 = \frac{2}{4} : 2 + \frac{6}{8} : 2$$

$$\left(\frac{2}{4} + \frac{6}{8}\right) \times \frac{1}{2} = \frac{2}{4} \times \frac{1}{2} + \frac{6}{8} \times \frac{1}{2}$$



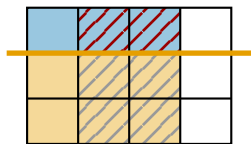
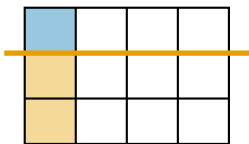
$$\left(\frac{6}{8} - \frac{2}{4} \right) : 2 = \frac{6}{8} : 2 - \frac{2}{4} : 2$$

$$\left(\frac{6}{8} - \frac{2}{4} \right) \times \frac{1}{2} = \frac{6}{8} \times \frac{1}{2} - \frac{2}{4} \times \frac{1}{2}$$



$$\left(\frac{1}{2} + \frac{1}{4}\right) : \frac{3}{2} = \frac{1}{2} : \frac{3}{2} + \frac{1}{4} : \frac{3}{2}$$

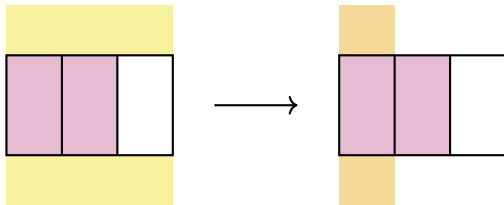
$$\left(\frac{1}{2} + \frac{1}{4}\right) \times \frac{2}{3} = \frac{1}{2} \times \frac{2}{3} + \frac{1}{4} \times \frac{2}{3}$$



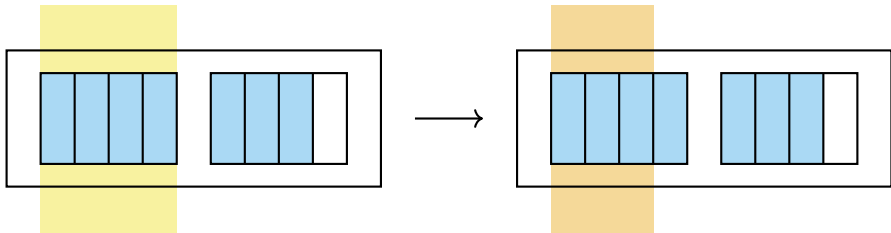
$$\left(\frac{1}{2} - \frac{1}{4}\right) : \frac{3}{2} = \frac{1}{2} : \frac{3}{2} - \frac{1}{4} : \frac{3}{2}$$

$$\left(\frac{1}{2} - \frac{1}{4}\right) \times \frac{2}{3} = \frac{1}{2} \times \frac{2}{3} - \frac{1}{4} \times \frac{2}{3}$$

On the result (dividing by a proper fraction)

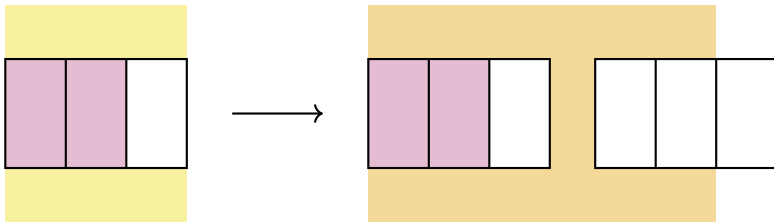


$$\frac{2}{3} < \frac{2}{3} : \frac{1}{3}$$

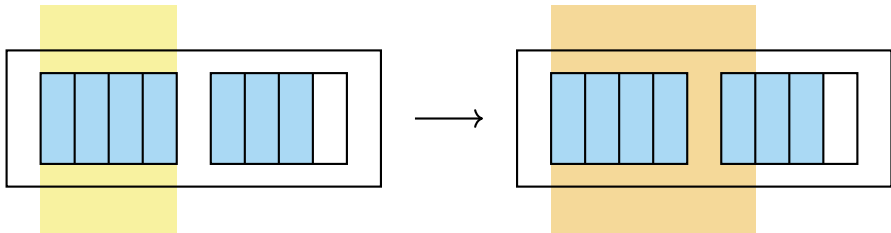


$$\frac{7}{4} < \frac{7}{4} \div \frac{3}{4}$$

On the result (dividing by an improper fraction)

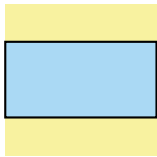


$$\frac{2}{3} > \frac{2}{3} : \frac{5}{3}$$

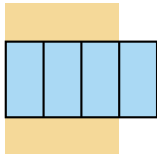
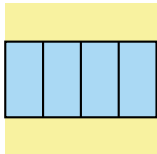


$$\frac{7}{4} > \frac{7}{4} \div \frac{5}{4}$$

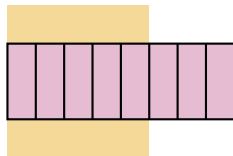
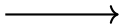
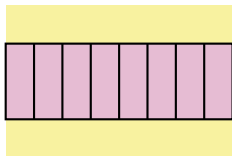
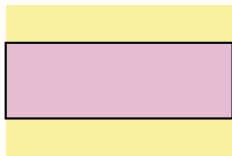
Taking the reciprocal



$$1 : \frac{3}{4} = \frac{4}{3}$$

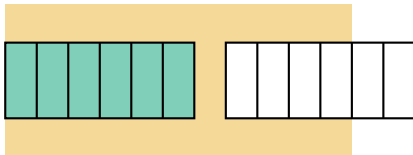
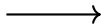
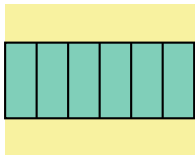


$$\frac{3}{4} \begin{array}{c} \curvearrowright \\ \curvearrowleft \end{array} \frac{4}{3}$$



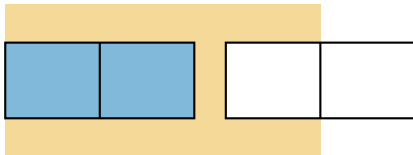
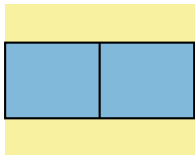
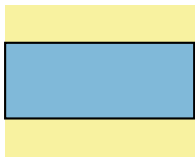
$$1 : \frac{5}{8} = \frac{8}{5}$$

$$\frac{5}{8} \quad \frac{8}{5}$$
Two curved arrows crossing each other, pointing from the 5 in the first fraction to the 5 in the second, and from the 8 in the first to the 8 in the second.



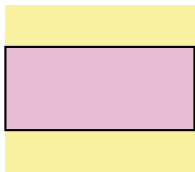
$$1 : \frac{10}{6} = \frac{6}{10}$$

$$\frac{10}{6} \quad \begin{array}{c} \nearrow \\ \searrow \end{array} \quad \frac{6}{10}$$

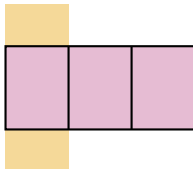
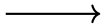
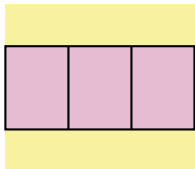


$$1 : \frac{3}{2} = \frac{2}{3}$$

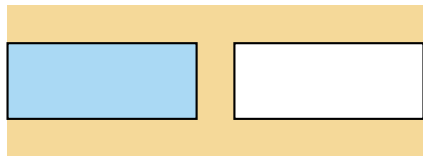
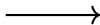
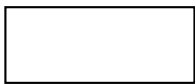
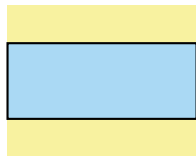
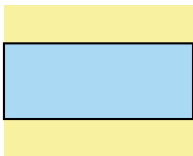
$$\frac{3}{2} \begin{matrix} \nearrow \\ \searrow \end{matrix} \frac{2}{3}$$



$$1 : \frac{1}{3} = 3$$



$$\frac{1}{3} \begin{matrix} \nearrow \\ \searrow \end{matrix} \frac{3}{1}$$



$$1 : 2 = \frac{1}{2}$$

$$\frac{2}{1} \begin{matrix} \nearrow \\ \searrow \end{matrix} \frac{1}{2}$$