

# Galois Representations, Automorphic Forms and their L-Functions

Closing conference of the ANR/FNR network GALF

**Dates:** 1-5 July 2024

**Place:** Belval Campus, University of Luxembourg

**Organisers:** Denis Benoist, Mladen Dimitrov, Alexandre Maksoud, Gabor Wiese

## Monday 1 July 2024

9.00-10.00 *Welcome with coffee and registration in front of MSA 3.370*

10.00-11.00 Fred Diamond: Geometric Serre weights and Jochnowitz modules for Hilbert modular forms (mod  $p$ )

11.15-12.15 Stefano Morra: On the smooth  $GL_2(Q_p)$ -representations in the mod  $p$  cohomology of Shimura curves

*lunch break*

14.00-15.00 Adel Betina: Eigenvarieties for non-cuspidal automorphic forms over PEL Shimura varieties

15.30-16.30 Chi-Yun Hsu: Eigenvariety for partially classical Hilbert modular forms

## Tuesday 2 July 2024

09.30-10.30 Luis Palacios: About the local geometry of the Bianchi eigenvariety around non-cuspidal points

11.00-12.00 Tobias Berger: Pseudomodularity of residually reducible Galois representations

*lunch break*

14.00-15.00 Andreea Iorga: Murphy's Law for Galois Deformation Rings

15.30-16.30 Loïc Merel: On the Harris-Venkatesh conjecture for modular forms of weight one

16.40-17.00 (Short talk) Sudhir Pujahari: A central limit theorem for sums of Hecke eigenvalues

## Wednesday 3 July 2024

09.30-10.30 Gergely Zarándi: Multivariable  $(\phi, \Gamma)$ -modules and extensions of principal series

11.00-12.00 Rebecca Bellovin: Local  $\varepsilon$ -isomorphisms in mixed characteristic

*lunch break*

14.00 *Guided tour of the Blast Furnace and the City of Science (in English or French)*

## Thursday 4 July 2024

09.30-10.30 Andrew Graham: Unitary Friedberg–Jacquet periods and anticyclotomic  $p$ -adic L-functions

11.00-12.00 Marco Sangiovanni-Vicentelli: Euler Systems: beyond motivic constructions

*lunch break*

14.00-15.00 Fabian Januszewski: Locally algebraic representations and the cohomology of arithmetic groups

15.30-16.30 Isabella Negrini: A Shintani lift for rigid cocycles

*Evening: Conference dinner*

## Friday 5 July 2024

09.00-10.00 Jaclyn Lang: Counting congruences between modular forms

10.30-11.30 Andrei Jorza:  $p$ -adic L-function for  $GL(4)$

*lunch break & end of conference*

12.45 PhD defence of Bryan Advocaat (MSA 3.370)

All talks take place in the Maison du Savoir (MSA), auditorium 3.370, on Belval Campus in Esch-sur-Alzette. Participants can use the class rooms MSA 3.380 and MSA 3.390 for discussions and individual work.

## Abstracts

### Rebecca Bellovin: Local $\varepsilon$ -isomorphisms in mixed characteristic

Given a representation of  $\mathrm{Gal}_{\{Q_p\}}$  with coefficients in a  $p$ -adically complete local ring  $R$ , Fukaya and Kato have conjectured the existence of a canonical trivialization of the determinant of a certain cohomology complex. Nakamura extended this conjecture to the setting of  $(\varphi, \Gamma)$ -modules over the Robba ring, and proved it in the rank-1 and trianguline settings. I will discuss how to extend these trivializations to certain families in mixed characteristic. This is joint work in progress with Ashwin Iyengar.

### Tobias Berger: Pseudomodularity of residually reducible Galois representations

After a survey of previous work I will present new results on pseudomodularity of residually reducible Galois representations with 3 residual pieces. I will discuss applications to proving modularity of Galois representations arising from abelian surfaces and Picard curves. This is joint work with Krzysztof Klosin (CUNY).

### Adel Betina: Eigenvarieties for non-cuspidal automorphic forms over PEL Shimura varieties

We construct Eigenvarieties parametrizing locally analytic overconvergent  $p$ -adic automorphic forms including Eisenstein families for unitary and symplectic groups. This is achieved by refining the method of Andreatta–Iovita–Pilloni for cuspidal automorphic forms and a deep study of the Hodge–Tate period map at the boundary of a toroidal compactification. The main novelty is the construction of subsheaves of the  $p$ -adic automorphic sheaves of Andreatta–Iovita–Pilloni by imposing a condition on the sections at the boundary. This talk is based on a joint work with Brasca–Rosso.

### Fred Diamond: Geometric Serre weights and Jochenowicz modules for Hilbert modular forms (mod $p$ )

I'll explain the statement of a Serre weight conjecture for Hilbert modular forms in characteristic  $p$ , viewed as sections of line bundles on Hilbert modular varieties (and so includes, for example, forms of partial weight one). I'll also discuss partial results towards it, and explain the statement of a refinement that captures additional structure in this context. This is joint work partly with Sasaki, and partly with Kassaei and Wiersema.

### Andrew Graham: Unitary Friedberg–Jacquet periods and anticyclotomic $p$ -adic $L$ -functions

Unitary Friedberg–Jacquet periods are a unitary group analogue of the linear periods studied by Friedberg–Jacquet, and are expected to satisfy a “Waldspurger-style relation” with square-root central critical  $L$ -values of automorphic representations. In this talk I will describe the construction of an anticyclotomic  $p$ -adic  $L$ -function interpolating these periods and discuss work in progress on relating special values of this  $p$ -adic  $L$ -function with split anticyclotomic Euler systems.

### Chi-Yun Hsu: Eigenvariety for partially classical Hilbert modular forms

Let  $F$  be a totally real field. A Hilbert modular form is a section of a modular sheaf, defined over the whole Hilbert modular variety associated to  $F$ , while a  $p$ -adic overconvergent form is defined only over a strict neighborhood of the ordinary locus. For each subset  $I$  of the primes of  $F$  above  $p$ , one has the intermediate notion of  $I$ -classical Hilbert modular forms by replacing ordinary by  $I$ -ordinary. Andreatta–Iovita–Pilloni constructed the Hilbert eigenvariety from their  $p$ -adic families of modular sheaves. I will talk about the construction of the eigenvariety of  $I$ -classical Hilbert modular forms following their approach. Potential applications are the study of partially weight one forms and the construction of partially de Rham Galois representations from partially classical Hilbert modular forms. This is a joint work in progress with Mladen Dimitrov.

### Andreea Iorga: Murphy's Law for Galois Deformation Rings

In this talk, I will prove that, under a specific assumption, any semi-direct product of a  $p$ -group  $G$  with a group  $\Phi$  of order prime-to- $p$  can appear as the Galois group of a tower of extensions  $M/L/K$  with the property that  $M$  is the maximal  $p$ -extension of  $L$  that is unramified everywhere, and  $\mathrm{Gal}(M/L) = G$ . A consequence of this is that any local ring admitting a surjection to  $\mathbb{Z}_5$  or  $\mathbb{Z}_7$  with finite kernel can be written as a universal everywhere unramified deformation ring.

## Fabian Januszewski: Locally algebraic representations and the cohomology of arithmetic groups

In this talk, I will discuss the notion of a locally algebraic representation in the context of the cohomology of arithmetic groups. Locally algebraic representations and their associated sheaves are of relevance for investigating integral structures and special values of cohomological automorphic representations, as well as related period relations. Furthermore, we will delve into concepts such as locally algebraic  $(g, K)$ -modules and cohomological types of automorphic representations, shedding light on their significance in this context. As an application, I will discuss the existence of canonical integral structures and the construction of canonical periods.

## Andrei Jorza: $p$ -adic L function for $GL(4)$

Recently, a lot of progress has been made in constructing  $p$ -adic L-functions for cuspidal representations of  $GL(2n)$  admitting Shalika models, with deep implications for the arithmetic of Galois representations. In work with Mladen Dimitrov, we construct the first general finite slope examples of  $p$ -adic families where the representation is allowed to have ramification at  $p$ . As a particular application, we construct  $p$ -adic families for all cuspidal representations of  $GL(4)$  which arise as the functorial transfer from  $GSp(4)$  and have parahoric level at  $p$ .

## Jaclyn Lang: Counting congruences between modular forms

For a given eigenform, how many other eigenforms of the same weight and level are congruent to it modulo a prime  $l$ ? We will explain how modular representation theory can be used to help answer this question. We focus especially on the case when the level is the square of a prime  $p$  such that  $p \equiv -1 \pmod{l}$ . This is ongoing joint work with Robert Pollack and Preston Wake.

## Loïc Merel: On the Harris-Venkatesh conjecture for modular forms of weight one

(with E. Lecouturier) This is part of the vast world of (a variant of) Venkatesh's conjectures about derived Hecke algebras. The situation of modular forms of weight one considered by Harris and Venkatesh might be the simplest significant case. Consider the one-to-one correspondence between: Newforms  $f$  of weight one and two-dimensional, complex, odd, irreducible representation  $\rho$  of the absolute Galois group. Consider such  $f$  and  $\rho$  with coefficients in a subring  $A$  of  $\mathbb{C}$ . Let  $q$  be a prime number unramified for  $\rho$  (equivalently, not dividing the level of  $f$ ). Consider the multiplicative group  $F_{q^*}$  of the finite field  $F_q$ . Harris and Venkatesh attach, up to sign, two elements in the tensor product of  $F_{q^*}$  and  $A$ :

- From  $f$ , without knowing  $\rho$ , a pseudo-eigenvalue of the derived Hecke operator at  $q$ ,
- From  $\rho$ , without knowing  $f$ , the localisation at  $q$  of a global cohomology class well defined up to a scalar in  $A$ .

The conjecture asserts that the second collection is equal to the first, up to an element of  $A$ . We will focus on the ways to determine the pseudo-eigenvalue of the derived Hecke operator at  $q$  using the Borisov-Gunnells theory.

## Stefano Morra: On the smooth $GL_2(\mathbb{Q}_p)$ -representations in the mod $p$ cohomology of Shimura curves

Let  $p$  be a prime number and  $K$  a finite extension of  $\mathbb{Q}_p$ . The mod  $p$  Langlands program for  $GL_2$  predicts the existence of finite length smooth mod  $p$  representations of  $GL_2(K)$  which are "naturally" attached to 2-dimensional mod  $p$  representations of the absolute Galois group of  $K$ , in particular which should appear in Hecke eigenspaces of the cohomology of Shimura curves with infinite level at  $p$ . This program has been realized for  $K=\mathbb{Q}_p$  in the early 2000, but it remains mysterious since almost 20 years as soon as  $K \neq \mathbb{Q}_p$ .

In this talk we show that, when  $K$  is unramified, the Hecke eigenspaces mentioned above produce finite length smooth mod  $p$  representation of  $GL_2(K)$ , whose structure is moreover predicted by the restriction at the decomposition group at  $p$  of the Galois representation attached to the Hecke eigensystems.

This is joint work in progress with C. Breuil, F. Herzig, Y. Hu et B. Schraen.

## Isabella Negrini: A Shintani lift for rigid cocycles

Rigid cocycles were defined in 2017 by Darmon and Vonk and give a promising framework to extend the theory of complex multiplication to real quadratic fields. They share striking parallels with modular forms, and their generalizations are the main ingredient in the emerging  $p$ -adic Kudla program.

In previous work I showed how to build a map from half-integral weight modular forms to rigid cocycles in the style of the Shimura lift. In this talk I will show how to construct a map going in the opposite direction, in the style of the Shintani lift.

**Luis Palacios:** About the local geometry of the Bianchi eigenvariety around non-cuspidal points

Let  $K$  be an imaginary quadratic field and  $p$  be a prime number. The Bianchi eigenvariety is a rigid analytic space that parametrizes (overconvergent, finite slope)  $p$ -adic automorphic forms for  $GL_2/K$ . In this talk, I will report on a work with Daniel Barrera on the local geometry of the Bianchi eigenvariety around certain non-cuspidal classical points, in particular, non-cuspidal base change points.

**Sudhir Pujahari:** A central limit theorem for sums of Hecke eigenvalues

In this talk, using recent developments in the theory of  $l$ -adic Galois representations, we will discuss an Erdős-Kac kind theorem for sums of eigenvalues of Hecke operators. Moreover, we will see an all-purpose Erdős-Kac theorem. In addition, we will discuss several applications to our main result. This is joint work with R. Murty and K. Murty.

**Marco Sangiovanni-Vicentelli:** Euler Systems: beyond motivic constructions

Euler Systems have proven to be versatile tools for understanding Selmer groups and their connections to special values of  $L$ -functions. However, despite their importance in foundational conjectures in number theory like the Bloch-Kato conjecture, only a handful of provably non-trivial Euler systems have been constructed to date.

A significant obstacle in constructing Euler Systems lies in producing candidate Galois cohomology classes. I will present a method to overcome this obstacle without relying on rare motivic classes like modular units or special cycles. I will survey joint work with C. Skinner in which we use Eisenstein classes on Siegel three-folds to construct a cyclotomic Euler System for the adjoint of an elliptic modular form.

**Gergely Zabradi:** Multivariable  $(\phi, \Gamma)$ -modules and extensions of principal series

Recently, several multivariable generalizations of Fontaine's  $(\phi, \Gamma)$ -modules have been introduced in order to generalize the  $p$ -adic local Langlands correspondence from  $GL_2(\mathbb{Q}_p)$  to groups of higher rank. In this talk we consider the notion that corresponds to representations of products of Galois groups via a Fontaine-style equivalence of categories. Further, there is a functor from the category of smooth admissible  $\text{mod } p^n$  representations of  $GL_n(\mathbb{Q}_p)$  to projective limits of such multivariable  $(\phi, \Gamma)$ -modules. This functor is exact on successive extensions of principal series. In the talk we make predictions how such successive extensions should look like and prove some preliminary results towards this. Partly joint work in progress with G. Jakovác.