Quantization of moduli spaces of bundles and the mapping class group

by

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Course References.

[A3] - [A5] gives proofs of the main results presented in the course.

[AGL] gives the construction of the Hitchin connection in the case where one also considers the metaplectic correction.

[A6] and [AG] could be a starting point for a first reading, since they provide an overview of the results from [A3] - [A5].

The series [AU1] to [AU4] gives a proof that the $SU(n)$-Reshetikhin-Turaev TQFT can be obtained from Conformal field theory. Combining this with Laszlo's result [La], one gets that the representations obtained from the geometric quantization of the moduli space of flat $SU(n)$-connections on a surface is projectively the same as the $SU(n)$-Reshetikhin-Turaev quantum representations of the mapping class group.

The references [At], [B1], [BHMV1], [BHMV2], [M1], [M2], [RT1], [RT2], [Ro], [T] and [W1] are TQFT references.

For references on moduli space of flat connections and their algebraic geometric counter part of moduli space of semi-stable bundles see [AB], [NS1], [NS2] and references there in.

For discussion of the prequantum line bundle over moduli space see [RSW], [Fr] and also [DN] which computes the algebraic geometric Picard group of the moduli spaces of semi-stable bundles also in the singular case.

The Hitchin connection was constructed in [H] from the algebraic geometric point of view for $SU(n)$ and for general Lie groups in [Fal]. From the point of view of infinite dimensional reduction from the space of connections in [ADW] and from the differential geometric point of view in [A4]. See also [R1] for the abelian case.

For general references on Toeplitz operators we refer to [BdMG], [BdMS], [BMS], [Sch], [Sch1] and [Sch2] and references there in.

For references to the closely related conformal field theory please see [TUY] and [Se] and references there in.

For a general reference to geometric quantization please see [Wo] and the references in there.

[A1] contains a couple of unpublished results. One of them is the study of the geometric quantization of the n-dimensional torus with respect to general linear non-negative polarizations. In particular it is proved there that the space of distributional sections of the k'th power of any prequantum line bundle on the torus, which are covariant constant along the polarization is finite dimensional of dimension k^n. It also gives an explicit isomorphism between this space and the space of theta functions.

[A2] does Toeplitz operators very explicitly in the case of a flat torus and provides an explicit global trivialization of the formal Hitchin Connection in this special case.

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