GEOQUANT 2013

Conference, August 26 – 30

1. **Anton Alekseev, Geneva, Switzerland**

   Title: Logarithms and deformation quantization (tentatively)

2. **Ugo Bruzzo, Trieste, Italy**

   Title: Stacky resolutions of moduli spaces of instantons

   Abstract: Moduli spaces of framed sheaves on the complex projective plane are a resolution of singularities of the moduli space of ideal instantons on the 4-sphere. Another instance of this phenomenon is provided by the moduli space of framed sheaves on the blowup of the projective plane, which resolves the singularities of the moduli space of ideal instantons on the orientation-reversed projective plane. A natural question arises whether similar constructions can be made in the case of other spaces. I will consider instantons on the ALE spaces X of type Ak. Here the nontrivial behaviour of the instantons at infinity makes it necessary to consider moduli spaces of framed sheaves on a DM projective stack whose coarse moduli space is a toric compactification of X. If time allows I will show how this construction fits naturally into the instanton counting over ALE spaces.

3. **Detlev Buchholz, Göttingen, German**

   Title: The resolvent algebra: a new approach to canonical quantum systems

   Abstract: The standard C^*-algebraic version of the algebra of canonical commutation relations, the Weyl algebra often causes difficulties. These stem from its failure to admit the formulation of physically interesting dynamical laws as automorphism groups, its lack of pysically significant observables and its unmanageable representation theory. In this talk a C^*-algebra of the canonical commutation relations is presented which circumvents such problems. It is based on the resolvents of the canonical operators and their algebraic relations. The resulting C*-algebra, the resolvent algebra, has many desirable analytic properties. In fact, it is of type I (postliminal) for finite quantum systems and nuclear in the infinite case. Its properties entail the existence of an abundance of one-parameter automorphism groups corresponding to physically relevant dynamics and of interesting observables. They are also useful in the discussion of supersymmetry, systems with constraints and the representation theory of Lie algebras of derivations. Moreover, the resolvent algebra has a rich and interesting ideal structure which encodes information about the dimension of the underlyng symplectic space. It thus provides an excellent framework for the rigorous analysis of finite and infinite quantum systems. (Joint work with Hendrik Grundling)

4. **Miroslaw Englis, Prague**

   Title: Spectral triples and Toeplitz operators

   Abstract: We use the machinery of generalized Toeplitz operators of Boutet de Monvel and Guillemin to construct examples of spectral triples (in the sense of A. Connes) based on the Berezin-Toeplitz quantization. [Joint work with B. Iochum and K. Falk.]

5. **Jürgen Fuchs Karlstad, Sweden**

   Title: Three-dimensional topological field theories on manifolds with boundaries and defects

   Abstract: We describe a bicategorical framework for topological boundary conditions and topological surface defects in three-dimensional topological field theories of Reshetikhin-Turaev type. Relevant tools include modules over tensor categories, the notion of center of a fusion category, central functors and the Witt group of modular tensor categories. We also outline potential applications to two-dimensional rational conformal field theories.

6. **Hajimi Fujita, Tokyo, Japan**

   Title: Equivariant local index and transverse index for circle action

   Abstract: In our joint work with Furuta and Yoshida we gave a formulation of index theory of Dirac-type operator on open Riemannian manifolds. We used a torus fibration and a perturbation by Dirac-type operator along fibers. In this talk we develop an equivariant version for circle action and apply it for Hamiltonian circle action case. We also investigate the relation between our equivariant index and index of transverse elliptic operator/symbol developed by Atiyah, Paradan-Vergne and Braverman.

7. **Tomohiro Fukaya, Sendai, Japan**

   Title: The coarse Baum-Connes conjecture for relatively hyperbolic groups

   Abstract: We study a group which is hyperbolic relative to a finite family of infinite subgroups. We show that the group satisfies the coarse Baum-Connes conjecture if each subgroup belonging to the family satisfies the coarse Baum-Connes conjecture and its classifying space is realized by a finite simplicial complex. We also construct a boundary of relatively hyperbolic group and show that its K-homology is isomorphic to the K-theory of the Roe algebra of the group under suitable assumptions. We give an explicit computation for the case of a non-uniform lattice of rank one symmetric space. This talk is based on the joint work with Shin-ichi Oguni.

8. **Alexey Gorodentsev, Moscow, Russia**

   Title: Mukai Lattices

   Abstract: Mukai lattice is a free $\mathbb{Z}$-module M equipped with unimodular (maybe nether symmetric nor skew symmetric) integer bilinear form. Mukai lattices that admit exceptional basis (i.e. a basis whose Gram matrix is upper triangular with units on the main diagonal) play especially important role. They appear in the algebraic geometry as K_0-groups of Fano varieties having exceptional basis in the derived category. They appear in the theory of singularities as vanishing cohomologies equipped with distinguished basis and the Seifert form. They appear in connection with quantum cohomologies and Frobenius manifolds as the Stokes matrices of semisimple Frobenius manifolds. There is a number of conjectures explaining the reasons for this phenomenon and linking the corresponding mathematical areas.

   We will discuss these links, these conjectures, old and new results obtained here, as well as (rather mysterious) connection of Mukai lattices with classical problems of the theory of Diophantine approximations:

It is known that the Mukai lattice $K_0(\mathbb{P}_2)$, of the projective plane, is closely connected with the Markov chain of homogeneous quadratic forms $f_m(x,y) \in \mathbb{Z}[x,y]$ of positive discriminant $D_m > 0$ such that $\min_{(x,y)\in\mathbb{Z}\setminus 0}\left(f(x,y)/\sqrt{(D)}\right)$ reach his absolute maximum among all the forms f at $SL_2(\mathbb{Z})$-orbit of $f = f_1$, then reach his absolute maximum among all the remaining forms f at $SL_2(\mathbb{Z})$-orbit of $f = f_2$, e.t.c. Namely, the Markov forms appear as restrictions of (the symmetric part of) the Mukai form on $K_0(\mathbb{P}_2) = \mathbb{Z}^3$ to the ortogonal planes to exceptional vectors (i.e. the vectors that can be included into some exceptional basis). Has this story a continuation to totally real cubic forms and $K_0(\mathbb{P}_3)$? Here even the whole chain of forms is unknown, only the first 2 forms were found by Davenport...

9. **Brian Hall, Notre Dame, USA**

   Title: The large-N limit of the Segal-Bargmann transform on unitary groups

   Abstract: I will describe recent results on the large-N limit of the Segal-Bargmann transform on the unitary group U(N). In the large-N limit, the heat kernel measure on U(N) concentrates onto a single conjugacy class. Thus, the transform on class functions becomes trivial in the limit. One can, however, extend the transform to matrix-valued functions and study functions that are equivariant under conjugation. In the limit, such functions concentrate to polynomials in a single U(N) variable. Thus, the limiting transform can be understood as a map of polynomials to polynomials, which we can compute explicitly. This is joint work with Bruce Driver and Todd Kemp of UCSD and is motivated by earlier work of Philippe Biane. Related results were obtained independently by Guillaume Cbron.

10. **Alexander Karabegov, Abilene, USA**

    Title: On Gammelgaards formula for a star product with separation of variables

    Abstract: We show that Gammelgaards explicit formula expressing a general star product with separation of variables on a Kaehler manifold in terms of directed acyclic graphs is equivalent to an inversion formula for an operator on a formal Fock space. We construct an algebra of operators containing this operator and its inverse. The operators in this algebra are expressed in terms of directed acyclic graphs. We give a composition formula for this algebra and then prove the inversion formula directly, thus offering an alternative proof of Gammelgaards formula which offers more insight into the question why the directed graphs in his formula have no cycles. Also, an explicit invariant formula for a star product with separation of variables can be easily deduced from Gammelgaard's formula.

11. **Will Kirwin, Cologne, Germany**

    Title: Complex-time evolution in geometric quantization

    Abstract: Using ideas of Thiemann, the adapted complex structure on a tubular neighborhood of a compact real-analytic Riemannian manifold in its tangent bundle can be interpreted as the "time-i" geodesic flow. On the tangent bundle of a compact Lie group, the composition of the geometric quantizaiton of this imaginary-time geodesic flow with the Schroedinger quantization of the associated hamiltonian yields Hall's coherent state transform (CST). In this talk, I will explain these ideas, and how they can be gener-

alized to yield an infinite-dimensional family of "complex-time" flows and associated generalized CSTs.

12. **Ryoichi Kobayashi, Nagoya, Japan**

    Title: Hamiltonian Volume Minimizing Property of Maximal Torus Orbits in Complex Projective Spaces

    Abstract: We prove that any $U(1)^n$-orbit in $\mathbb{P}^n$ is Hamiltonian volume minimizing, i.e., volume minimizing under Hamiltonian deformation. The idea is the following :

    (1) We extend one $U(1)^n$-orbit to the moment torus fibration $\{T_t\}_{t\in\Delta_n}$ and consider its Hamiltonian defoemation $\{\phi(T_t)\}_{t\in\Delta_n}$ where ϕ is a Hamiltonian diffeomorphism of $\mathbb{P}^n$ and then :

    (2) We consider each $U(1)^n$-orbit and its Hamiltonian deformation by comparing the large k asymptotic behavior of the sequence of projective embeddings defined, for each k, by the basis of $H^0(\mathbb{P}^n, \mathcal{O}(k))$ obtained by semi-classical approximation of the $\mathcal{O}(k)$-BS (Bohr-Sommerfeld) tori of the Lagrangian torus fibration $\{T_t\}_{t\in\Delta_n}$ and its Hamiltonian deformation $\{\phi(T_t)\}_{t\in\Delta_n}$.

13. **Gandalf Lechner, Leipzig, Germany**

    Title: Non-local perturbations of hyperbolic PDEs and QFT models on non-commutative spacetimes

    Abstract: In the context of formulating quantum field theories over non-commutative spacetimes, such as Moyal Minkowski space or locally non-commutative versions thereof, one is led to consider a class of partial differential equations of the form (D+W)f=0, where D is a normally hyperbolic differential operator or Dirac operator, and W is some non-local perturbation ("non-commutative potential"). Despite the non-locality, we construct advanced and retarded fundamental solutions for such PDEs, discuss the structure of their solutions, and compute the scattering behaviour. These systems can be quantized, and a non-commutative field theory can be extracted by a version of Bogoliubov's formula. We shall also point out the similarities and differences to other approaches to QFT on non-commutative spacetimes. Joint work with Markus Borris and Rainer Verch.

14. **Xiaonan Ma, Paris, France**

    Title: Flat vector bundle and Toeplitz operators

    Abstract: The real analogue of the Dolbeault complex is the de Rham complex associated with a flat vector bundle. In this talk, we explain first the theory of Toeplitz operators associated with a bundle positive lines, and how the Toeplitz operators in complex geometry appear naturally in the study of the asymptotic properties of cohomology or spectrum associated with a family of flat vector bundles F_p. For example, a real version of the vanishing theorem of le Potier is established using properties of the Toeplitz operator. We can give also the asymptotic informations for the size of the torsion elements in the cohomology group in the arithemetic case.

15. **George Marinescu, Cologne, Germany**

    Title: Equidistribution of random zeros on complex manifolds

    Abstract: This talk is concerned with the almost sure convergence to an equilibrium distribution of zeros of random sequences of holomorphic zero sets . For example, the zeros of SU(2) polynomials studied in the physics literature tend to accumulate to a uniform distribution, the Fubini-Study measure on the Riemann sphere. The equidistribution follows in general from the asymptotic expansion of the Bergman kernel, which gives an expansion of the expected values of the zero-currents. This talk is based on joint work with D. Coman, T.-C. Dihn and X. Ma.

16. **Shin-Ichi Oguni, Matsuyama, Japan**

    Title: On the coarse Baum-Connes conjecture

    Abstract: An assembly map encodes higher indexes of elliptic differential operators. The coarse Baum-Connes conjecture claims that two "coarse homology theories" for nice proper metric spaces are equivalent by a coarse version of the assembly map. The conjecture is known to be deeply related to differential topology of closed manifolds with infinite fundamental groups. In this talk, I will explain the above and introduce some known results containing the case of non-positively curved metric spaces in the sense of Busemann. This talk is based on a joint work with Tomohiro Fukaya.

17. **Alexey Parshin, Moscow, Russia**

    Title: Base change and automorphic induction in the Langlands theory

18. **Armen Sergeev, Moscow**

    Title: Quantum calculus and noncommutative Bloch theory

19. **Andrei Shafarevich, Moscow, Russia**

    Title: Quantization conditions on Riemann surfaces and spectral series of non-selfadjoint operators

    Abstract: Spectral series of a wide class of selfadjoint quantum operators satisfy Maslov quantization conditions on invariant Lagrangian submanifolds of the real classical phase space. According to theses conditions, integrals of the fixed 1-form over arbitrary cycle on the manifold must be integers. The selfadjointness of the operator implies that the series are semiclassically close to the exact spectrum. For non-selfadjoint operators the situation is completely different: the phase space is complex and spectral series can lie far from the exact spectrum. We study these problems for certain operators, appearing in hydrodynamics and magnetohydrodynamics; we show that in these cases asymptotics of the spectrum satisfies special quantization conditions on Riemannian surfaces - complex curves of constant energy; according to these conditions the integral of the fixed holomorphic 1-form over some (not arbitrary) cycle must be integer. The spectrum asymptotically converges to the points of a graph on the complex plane and different cycles of the surface correspond to different edges of this graph.

20. **Georgy Sharygin, Moscow, Russia**

Title: Full symmetric Toda system and Bruhat order

Abstract: The talk is based on a joint paper with Yu.Chernyakov (ITEP) and A.Sorin (JINR). We consider the full symmetric (nonperiodic) Toda chain. We show, that the stable points of this system are equal to the diagonal matrices with same eigenvalues, written in different order. We also show that two such matrices are connected by a trajectory, iff the corresponding permutations are compareble in Bruhat order on the group of permutations and trajectories in this case span a submanifold, whose dimension is determined by the lengths of the corresponding permutations, so that the system has a Morse-Smale structure (if we pass to suitable coordinates).

21. **Dimitry Talalaev, Moscow, Russia**

Title: On deformation quantization of integrable systems

Abstract: I'll talk about the problem of deformation of a pair: a Poisson algebra and its commutative subalgebra in the framework of *-product. For general pairs one can construct quantization obstructions and interpret them as some classes in relative Poisson cohomology of the relative Hochschild complex of a pair. I'll demonstrate that in a particular case these classes coincide with ones obtained earlier by Garay and Van Straten. The talk is based on a joint work with G. Sharygin arXiv:1210.2840.

22. **Alejandro Uribe, Michigan University, USA**

Title: The exponential map of the complexification of $Ham(M, \omega)$ in the real-analytic case.

Abstract: Let (M, ω, J) be a Kähler manifold and $\mathcal{G}$ the group of hamiltonian symplectomorphisms. The complexification of $\mathcal{G}$ introduced by Donadson is not a group -no such group exists- only a "formal Lie group". However it still makes sense to talk about the exponential map in the complexification. In this talk we describe how to construct geometrically the exponential map (generically only for small time), in case the initial data are real-analytic. The construction is motivated by, but does not use, semiclassical analysis.

23. **Michele Vergne, Paris, France**

Title: An Euler-MacLaurin formula for the equivariant index of a transversally elliptic operator

Abstract: I will report on a common work with De Concini and Procesi. Let G be a compact connected Lie group with maximal torus T acting on a manifold M. Our aim is to understand the qualitative nature of the multiplicities of representations of G constructed geometrically as indices of (transversally) elliptic operators P on M. Let σ be the principal symbol of P We use the infinitesimal index map to construct an antinivariant spline function on $Lie(T)^*$. Under some simplifying hypothesis, this spline function restricts to the lattice of weights of T as the multiplicity. This result can be seen as a generalization of the $[Q, R] = 0$ theorem.

24. **Siye Wu, Hong Kong, China**

Title: Hitchin's equations on a non-orientable manifold

Abstract: We study Hitchin's equations and Higgs bundles over a non-orientable manifold whose oriented cover is compact Kähler. Using the involution induced by the deck transformation, we show that the moduli space of Higgs bundles is Langrangian/complex with respect to the hyper-Kähler structure on Hitchin's moduli space associated to the oriented cover. We then establish a Donaldson-Corlette type correspondence and relate Hitchin's moduli space to representation varieties. This is a joint work with N.-K. Ho and G. Wilkin.

25. **Tilmann Wurzbacher, Metz, France/Bochum, Germany**

Title: Integration of vector fields on supermanifolds and applications

26. **Takahiko Yoshida, Tokio, Japan**

Title: On the local index

Abstract:

In the joint work with Fujita and Furuta we gave an index theory of Dirac-type operators on open Riemann manifolds provided that they have certain geometric structrues on their ends. In our work we used the idea of Witten's deformation for Dirac-type operators. In this talk we consider our index and the symplectic cutting for a Hamiltonian circle action. In particular, we give a formula for the index of the reduced space in a symplectic cut space.

27. **Xiangyu Zhou, Beijing, China**

Title: Some results on the L^2 extension problem