Talks on the Conference Geoquant 2019, Taiwan

September 9 – 13, 2019

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1. **Jorgen Ellegaard Andersen** (University of Aarhus, Aarhus)

   Title: General quantization schemes for symplectic manifolds which admit Kähler structures.

   Abstract:

   We will consider arbitrary holomorphic families of Kähler structures on a fixed prequantizable symplectic manifold and provided the corresponding family of Kähler quantisations for level k form a vector bundle over the chosen family of Kähler structures, we will for large enough k construct a large family of Hitchin connections in this bundle. We will also consider a natural family of Hermitian structures on the bundle of quantum spaces and understand which (unique) Hitchin connection is compatible with a given one of these Hermitian structures. Once we have selected either a Hermitian structure or a Hitchin connection, for the given family, we will provide a quantisation construction for an certain algebra of quantizable observables (sub-algebra of the real analytic functions) and a quantisation of a certain subset ofquantizableSpecial Lagrangian subvarieties, which are parallel w.r.t. to the chosen Hitchin connection. If we have a subgroup of the symplectomorphisms, which preserves the family of Kähler structures and whose action has been lifted to the given prequantum line bundle and the family Kähler structures is equivariant under this subgroup, then both the Hermitian structures and the Hitchin connections can be chosen such that they are invariant under this subgroup and the quantisation of both observables and special Lagrangians are equivariant under the action of this subgroup. The size of the algebra of quantizable observables and the set quantizable Special Lagrangian subvarieties depends on the size of the family of Kähler structures: the large the family is, the small the algebra of quantizable observables and the smaller the set of quantizable Special Lagrangian subvarieties is. All real analytic functions and all special Lagragians are quantizable if the family of Kähler structures is one single Kähler structure (which can be any on from the infinite dimensional space of all Kähler structures). If on the other extreme one considers the entire infinite dimensional family of Kähler structures on the symplectic manifold, then one will get Hermitian structures and corresponding Hitchin connections which are invariant under the full group of all symplectomorphisms of the symplectic manifold, but in this case there are almost no quatizable functions nor any quantizable special Lagrangians. We emphasise that how a given observable or a given special Lagrangian subvariety is quantised depends strongly on the chosen family of Kähler structures.

2. **Kwok Wai Chan** (Chinese University of Hong Kong, Hong Kong)

   Title: Asymptotic representation of Toeplitz quantization

   Abstract:

   In this talk, I will explain a construction of Fedosov's deformation quantization using a localized version of Toeplitz operators. This then gives an asymptotic action of the

deformation quantization algebra on the Hilbert spaces coming from geometric quantization. This is based on ongoing joint work with Naichung Conan Leung and Qin Li.

3. **Ivan Alekseyevich Dynnikov** (Steklov Institute of Mathematics, Moscow),

   Title: The dynamics of foliations on surfaces defined by closed 1-forms and conductivity

   Abstract: In the beginning of 1980's S.Novikov proposed to study the asymptotic behaviour of unbounded components of plane sections of 3-periodic surfaces in the 3-space. This geometrical problem arose in the conductivity theory for normal metals exposed to magnetic field, and gives rise to a family of measured oriented foliations on closed orientable surfaces. General foliations of this kind have been extensively studied during the last several decades in connection with interval exchange transformations and the theory of moduli spaces of Riemann surfaces. However, the family of foliations that arises in Novikov's problem is very special, and many natural questions about it are still open. The most important of them is to describe, for any given generic surface or a 1-parametric family of level surfaces of a fixed function, the set of so called chaotic regimes. I will overview the results concerning Novikov's problem including some recent updates.

4. **Mikhail Feigin** (University of Glasgow, Glasgow),

   Title: Dunkl angular momenta algebra

   Abstract:

   Dunkl operators are associated with a finite Coxeter group. They are deformations of partial first order derivatives by a combination of reflection operators. Together with polynomials and Coxeter group they generate rational Cherednik algebra. The algebra contains Dunkl Laplacian which is a deformation of the usual Laplace operator. It is closely related to Calogero-Moser integrable system.

   Dunkl angular momenta algebra is a subalgebra of the rational Cherednik algebra generated by Dunkl angular momenta and Coxeter group. It is a deformation of the skew product of the algebra of functions on the cone over Grassmanian of two-dimensional planes and the Coxeter group. Its centre is closely related to the angular part of the Calogero-Moser Hamiltonian. A version of Dunkl angular momenta algebra may also be thought of as the hidden symmetry algebra of the Dunkl Laplacian, in the same way as the universal enveloping algebra of so(n+1) is the hidden symmetry algebra of Coulomb problem in n-dimensional space. In fact the two situations can be unified via a Dunkl version of Laplace-Runge-Lenz vector.

   The talk is based on joint works with T. Hakobyan.

5. **Chin-Yu Hsiao** (Academia Sinica, Taipei)

   Title: Geometric quantization on CR manifolds

   Abstract:

   In this work, we establish "quantization commutes with reduction theorem" on CR manifolds. A big difference between CR setting and symplectic setting is that the spaces we considered in CR setting are infinite dimensional. This is a joint work with Xiaonan Ma and George Marinescu.

6. **Rung-Tzung Huang** (National Central University, Taoyuan)

   Title: G-invariant Szegö kernel asymptotics and CR reduction

   Abstract:

   Let $(X, T^{1,0}X)$ be a compact connected orientable CR manifold of dimension $2n+1$ with non-degenerate Levi curvature. Assume that X admits a connected compact Lie group action G. Under certain natural assumptions about the group action G, we show that the G-invariant Szegö kernel for $(0,q)$ forms is a complex Fourier integral operator, smoothing away $\mu^{-1}(0)$ and there is a precise description of the singularity near $\mu^{-1}(0)$, where μ denotes the CR moment map. We apply our result to the case when X admits a transversal CR S^1 action and deduce an asymptotic expansion for the m-th Fourier component of the G-invariant Szegö kernel for $(0,q)$ forms as $m \to +\infty$.

   As an application, we show that if m large enough, quantization commutes with reduction. This is a joint work with Chin-Yu Hsiao.

7. **Louis Ioos** (University of Tel Aviv, Tel Aviv)

   Title: Geometric quantization of symplectic maps and semi-classical trace formulas

   Abstract:

   We present a general construction of quantum maps associated to symplectic diffeomorphisms of a compact prequantized symplectic manifold, which recovers the Kostant-Souriau quantization of Hamiltonian flows on one hand and the quantum representations of mapping class groups on the other hand. Using the theory of Berezin-Toeplitz operators of Ma and Marinescu, we compute semi-classical asymptotics for their trace, then present applications to Gutzwiller's trace formula and Witten's asymptotic expansion conjecture.

8. **Atsushi Kanazawa** (University of Kyoto, Kyoto)

   Title: Tyurin degenerations and Lagrangian torus fibrations of Calabi-Yau manifolds

   Abstract:

   A Tyurin degeneration is a degeneration of a Calabi-Yau manifold to a union of two quasi-Fano manifolds intersecting along a common smooth anti canonical divisor. This is known to be an analogue of a Heegaard decomposition of a compact oriented 3-manifold into two handlebodies. I will discuss some interplay between Tyurin degenerations and Lagrangian torus fibrations, with particular emphasis on DHT and SYZ conjectures.

9. **William Elbaek Mistegaard** (University of Aarhus, Aarhus)

   Title: Resurgence analysis of quantum invariants

   Abstract:

   In this talk I will present joint work with Jorgen Ellegaard Andersen, which concerns the use of resurgence analysis in quantum topology. In the case of a Seifert fibered integral homology three sphere, our results include the following: 1) The quantum invariant associated with the special unitary group of rank two detects all Chern-Simons values of flat principal G bundles, where G is the complex special linear group of rank two. 2) This quantum invariant can be reconstructed from its asymptotic expansion, through a Borel-Laplace like summation.

10. **Martin Schlichenmaier** (University of Luxembourg, Luxembourg)

    Title: Some naturally defined star products for Kaehler manifolds

    Abstract:

    We give for the Kähler manifold case an overview of the constructions of some naturally defined star products. In particular, the Berezin-Toeplitz, Berezin, Geometric Quantization, Bordemann-Waldmann, and Karbegov standard star product are introduced. With the exception of the Geometric Quantization case they are of separation of variables type. The classifying Karabegov forms and the Deligne-Fedosov classes are given. Besides the Bordemann-Waldmann star product they are all equivalent.

11. **Armen Sergeev** (Steklov Mathematical Institute, Moscow)

    Title: On the harmonic spheres conjecture

    Abstract:

    The harmonic spheres conjecture establishes a correspondence between Yang–Mills G-fields on $\mathbb{R}^4$ and harmonic maps of the Riemann sphere S^2 into the loop space ΩG of the group G. It is an extension to general Yang–Mills G-fields of the Atiyah–Donaldson theorem establishing a correspondence between the moduli space of G-instantons on $\mathbb{R}^4$ and holomorphic maps $S^2 \to \Omega G$.

    In my talk I shall present an approach to the proof of this conjecture based on the adiabatic limit construction. Using this construction we can associate with an arbitrary Yang–Mills G-field on S^4 a harmonic map of the sphere S^2 to the loop space ΩG.

12. **Georgiy Sharygin** (ITEP, Moscow)

    Title: Geometry of full symmetric Toda system on compact groups

    Abstract:

    The full symmetric Toda system is a straightforward generalization of the usual (3-diagonal) system; it can be further generalized to the case of Cartan decomposition of an arbitrary real semisimple Lie group. In this case the integrability of the system is known, but the constructions of the involute families of integrals are usually quite complicated. In my talk I will describe a construction of commutative family of vector fields on the compact group, analogous to the family of first integrals in involution. This construction is based on the structure of representations of the original group. I will also describe the relation of this construction with Sorin and Chernyakov's and Reshetikhin and Schrader's constructions, proving the noncommutative integrability of the system.

13. **Oleg K. Sheinman** (Steklov Mathematical Institute, Moscow)

    Title: Separation of variables for Hitchin systems on hyperelliptic curves

    Abstract:

    Hitchin systems were invented in 1984 and since that time have been successfully applied in 2D CFT, geometric Langlands program and other questions. For Hitchin systems, the classical problems of the theory of integrable systems, like finding the action–angle coordinates, Lax representations, θ-functional formulae for solutions, have been addressed by K.Gawedzki (1998), A.Gorsky, N.Nekrasov, V. Rubtsov (2001), I.Krichever (2001). However, certain basic questions are still open. In the talk, we will give an explicit

description of the class of spectral curves of the Hitchin systems on hyperelliptic curves of all genera, for all classical simple complex groups. Based on this description, and on standard technique of separation of variables, we provide explicit expressions for the Darboux coordinates for those systems. The cases of $SL(2)$, $SO(4)$, $Sp(4)$ are assumed to be considered in more detail (earlier, only the $SL(2)$ case was explicitly described in 1998 by K.Gawedzki). The talk is based on the works

[1] O.K.Sheinman, *Spectral curves of the hyperelliptic Hitchin systems*, arXiv:1806.10178 [math-ph] (submitted to Func. Analysis and Appl.).

[2] P.I.Borisova, O.K.Sheinman, *Hitchin systems on hyperelliptic curves*, Proceedings of the Steklov Mathematical Institute, to be published.

[3] P.I.Borisova, *Separation of variables for the type D_l Hitchin systems on a hyperelliptic curve*, Russ. Math. Surv., to be published.

14. **Ryosuke Takahashi** (Kyoto University, Kyoto)

    Title: The inverse Monge-Ampère flow and applications to Kähler-Einstein metrics

    Abstract:

    We introduce the "inverse Monge-Ampère flow", a new parabolic flow which is designed to deform a given Kähler metric to a Kähler-Einstein one, and fits Donaldson's new GIT picture. We provide some convergence results for the inverse Monge-Ampère flow. This talk is based on a joint work with T. C. Collins (Harvard Univ.) and T. Hisamoto (Nagoya Univ.).

15. **Kenji Ueno** (Yokkaichi University, Yokkaichi)

    Title: Abelian conformal field theory and SL(2,C) conformal blocks on curves of genus 2

    Abstract:

16. **Katrin Wendland** (Albert-Ludwigs-University, Freiburg)

    Title: Folding Hitchin integrable systems and orbifolds

    Abstract:

    In 2006, Diaconescu, Donagi and Pantev found an intriguing relationship between Hitchin integrable systems and Calabi-Yau integrable systems, both associated to any complex ADE Lie group of adjoint type. The construction is centered around a family of quasi-projective Calabi-Yau threefolds for each choice of ADE Lie group. In his 2016 thesis, Beck generalized this construction to include the non-simply laced complex simple Lie groups, whose Dynkin diagrams are obtained from the diagrams of some of the simply-laced Lie groups by 'folding. In this talk, we report on recent joint work with Florian Beck and Ron Donagi, where we relate the folding procedure to orbifoldings in the above-mentioned correspondence between integrable systems.

17. **Akira Yoshioka** (Tokyo University of Sciene, Tokyo)

    Title: Convergent star products and star functions

    Abstract:

    Deformation quantization is to give an associative product of functions on a manifold by deforming the usual product of functions. It is constructed as a power series of

bidifferential operators. The product is called a star product. When the power series is a formal one, we know that on any manifold, there exists a deformation quantization of formal power series (Kontsevich). However, when we consider star products of non-formal power series, we know little about the products at present.

In this talk, we discuss non-formal star products on $\mathbb{C}^n$. We introduce certain topologies in the function space and discuss convergence and continuity of the products. Using convergent star products, we can construct star functions, which are deformation of usual entire functions such as the exponential fucntions. We show some examples of star functions.

18. **Steven Morris Zelditch** (Northwestern University, Evanston)

Title: Partial Bergman kernels and interfaces

Abstract:

Partial Bergman kernels are kernels of orthogonal projections to subspaces S_k of the space $H^0(M, L^k)$ of holomorphic sections of powers of an ample line bundle. When S_k is the spectral subspace of a Toeplitz Hamiltonian corresponding to an interval $[0, E]$ of eigenvalues, the partial density of states, suitably normalized, drops from 1 in the allowed region to 0 in the forbidden region. We show that the scaled kernel has Gaussian error asymptotics across a regular interface between the regions. When the interface is singular, the asymptotics are quite different. An especially interesting singular level is the minimum of the classical Hamiltonian, and we compare partial Bergman kernels at the Planck scale to ground states of the Hamiltonian.

Contributed talks:

1. **Jill Ecker** (University of Luxembourg, Luxembourg)

   Title: The low-dimensional algebraic cohomology of the Witt and the Virasoro algebra

   Abstract:

   The aim of our work is to compute the first and the second algebraic cohomology of the Witt and the Virasoro algebra with values in general tensor densities modules. We are interested in the full algebraic cohomology and not only the sub-complex of continuous cohomology, meaning we do not put any continuity constraints on the cochains. The talk will start with a short introduction to the Witt and the Virasoro algebra. In a second step, we will briefly recall the Chevalley-Eilenberg cohomology of Lie algebras with values in general tensor densities modules. In a third step, we will compute the first algebraic cohomology in order to provide a warm-up example and to illustrate our algebraic techniques. Finally, we will discuss the computation of the second algebraic cohomology, and we will also give some results concerning the third algebraic cohomology.

2. **Andrea Galasso** (Universita degli Studi di Milano-Bicocca, Milano)

   Title: Equivariant asymptotics of Szegö kernels

   Abstract:

   Assume that a compact Lie group G acts on a compact quantized manifold M in a Hamiltonian way. Suppose that this action defines a unitary representation on the Hilbert space H of the quantization. One is then led to study the asymptotic properties of the projectors onto the isotypical components associated to a weight $k\,\nu$ when $k \to +\infty$. If time permits, I will explain how to define Equivariant Toeplitz operators in this context.

3. **Chung-I Ho** (National Kaohsiung Normal University, Kaohsiung)

   Title: Nonorientable Lagrangian surfaces in rational 4-manifolds

   Abstract:

   The existence of Lagrangian submanifolds is an important problem in symplectic geometry. Most of the study has been focused on the orientable cases. In this talk, we will give topological and homological constraints for nonorientable surfaces in symplectic rational 4-manifolds.

4. **Laurent La Fuente-Gravy** (University of Luxembourg, Luxembourg)

   Title: Moment maps and Fedosov star products

   Abstract:

   I will describe a moment map on the space of symplectic connections on a given closed symplectic manifold. The value of this moment map at a symplectic connection is contained in the trace density of the Fedosov star product attached to this connection. Moreover, this Fedosov star product can only be closed when the symplectic connection lies in the vanishing set of the moment map.

I will give other examples of moment maps appearing in the trace density of star products. Motivated by this examples, I will present the classical obstructions to the existence of cscK metrics that one can adapt in the framework of symplectic connections.

5. **Hung-Yu Yeh** (Academia Sinica, Taipei)

   Title: Stability filtrations and weakly ample sequences in categories

   Abstract:

   Stability, first introduced by Mumford in the 1960's, is used as a tool to construct moduli space of sheaves on algebraic varieties and later generalized to objects in arbitrary abelian category. On the other hand, motivated by homological mirror symmetry conjecture and Douglas' Pi stability on the category of B-branes Bridgeland introduces stability conditions on triangulated categories which depends on the existence of Harder-Narasimhan (HN) filtration and central charges on the relevant K group of associated triangulated categories. In this talk, I would like to present main ideas in my current work and introduce a notion of stability filtration in arbitrary categories which is equivalent to the existence of HN filtration on objects. Indeed it is equivalent to existences of a zero morphism 0, a partial order on objects, and a collection of some universal sequences. One may give a suitable addition with this 0 morphism on the Hom space which makes this category an additive category. Then with weakly ample sequences in the additive category embedded in an ambient triangulated category under suitable conditions, we could obtain a numerical polynomial or central charge of objects by calculating the Euler characteristic of weakly ample sequences and objects, inducing a partial order and HN filtration. At the end, I would give some easy examples in algebraic curves.