Choosing visually balanced helices

How can we choose the parameters (r, c) for a visually balanced helix? We explore two ideas:

1. **Balanced curvature:** If $r = |c|$, the helix achieves maximal curvature for a given pitch[4]. For this combination, the helix is neither too "flat" (dominant r) nor too "stretched" (dominant c).

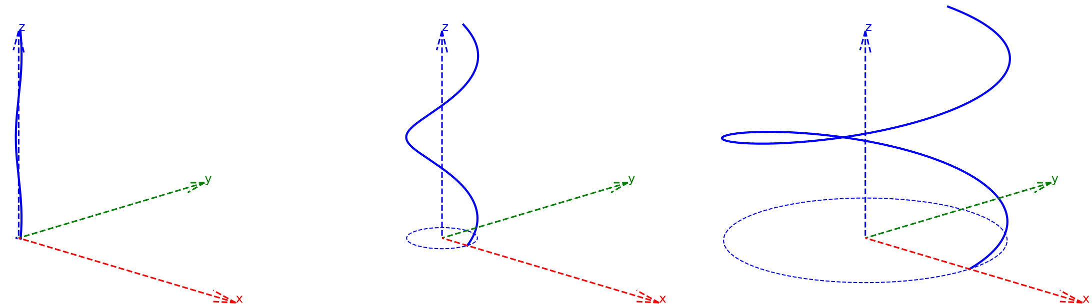

Figure 9: Three helices with radii $r = 0.01$, $r = c$ and $r = 0,5$ and the same constant c.

2. **Golden ratio:** If we have a ratio

$$\frac{r}{c} = \Phi \quad \text{with} \quad \Phi = \frac{1+\sqrt{5}}{2} \approx 1.618 \quad \text{(golden ratio)}$$

the helix aligns with a proportion often associated with aesthetic harmony in nature and art.

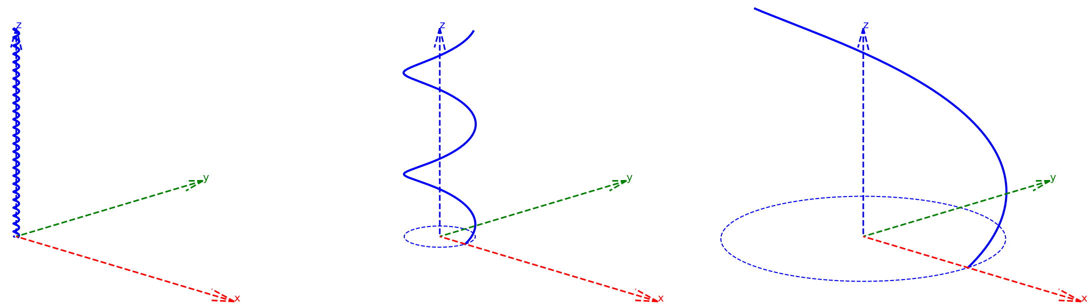

Figure 10: Three helices with radii $r = 0.01$, $r = 0,125$ and $r = 0,5$ and a constant $c = \frac{r}{\Phi}$.

Remark: We can also observe that if the ratio $\dfrac{r}{c} = \lambda$ (e.g. $\lambda = \Phi$) is a constant value, then all helices with parameters $(r, c = \frac{r}{\lambda})$ are geometrically similar up to rescaling. This can be seen by expressing the helix coordinates as:

$$\begin{pmatrix} r\cos(t) \\ r\sin(t) \\ ct \end{pmatrix} = \begin{pmatrix} r\cos(t) \\ r\sin(t) \\ \frac{r}{\lambda} \cdot t \end{pmatrix} = r \cdot \begin{pmatrix} \cos(t) \\ \sin(t) \\ \frac{1}{\lambda} \cdot t \end{pmatrix}$$

A similar argument applies if the ratio $\dfrac{c}{r}$ is constant.

[4] See *Resources > An intuitive definition of curvature* for details