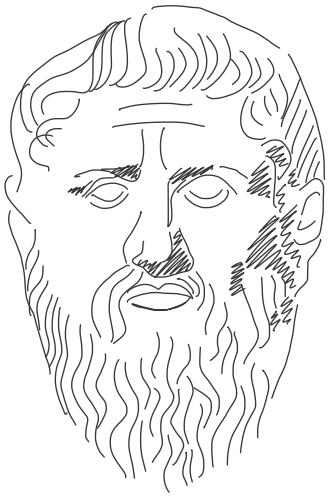
Random Multi-Geodesics on Hyperbolic Surfaces



Orsay 24.11.2022

joint work with

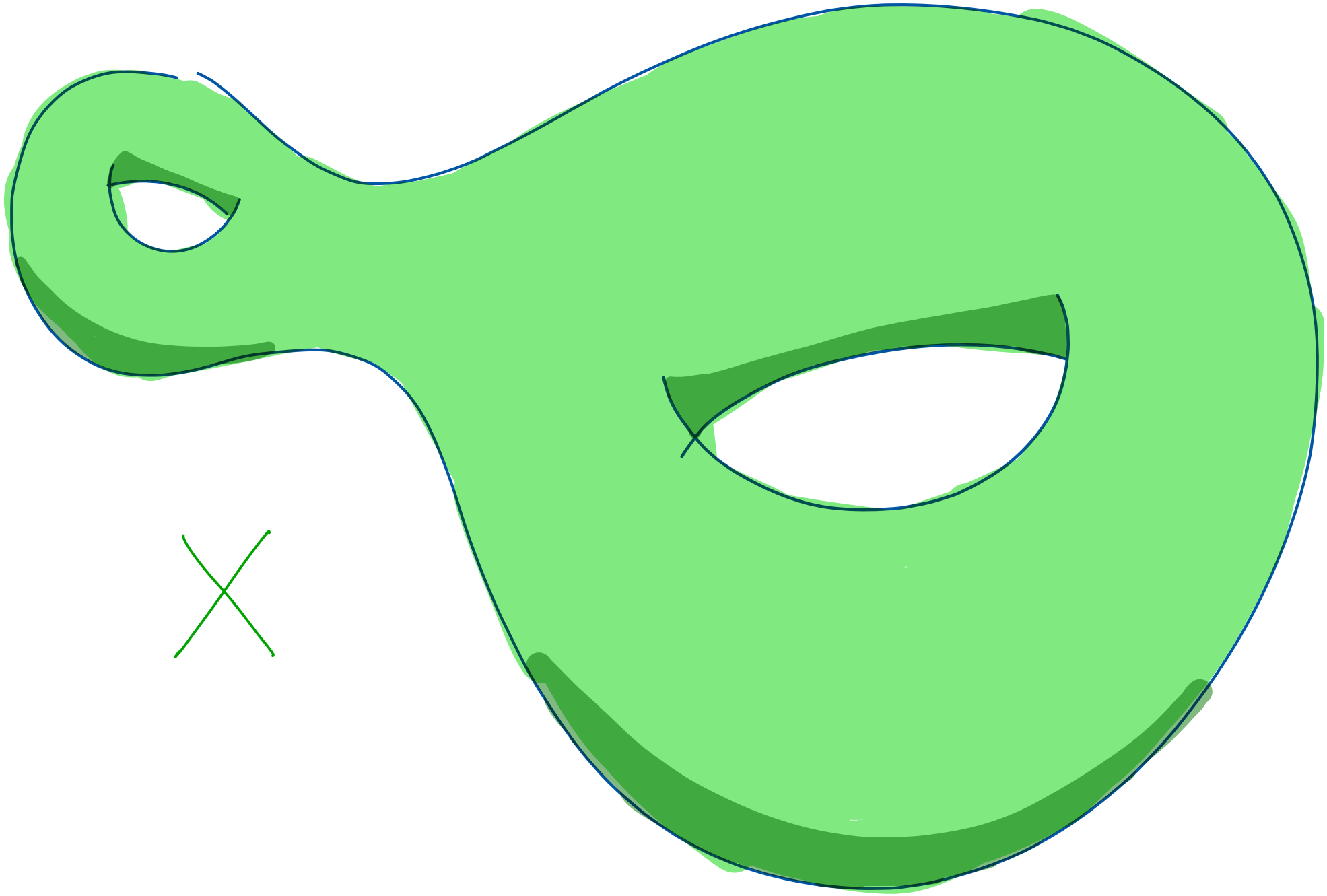
Vincent Delecroix

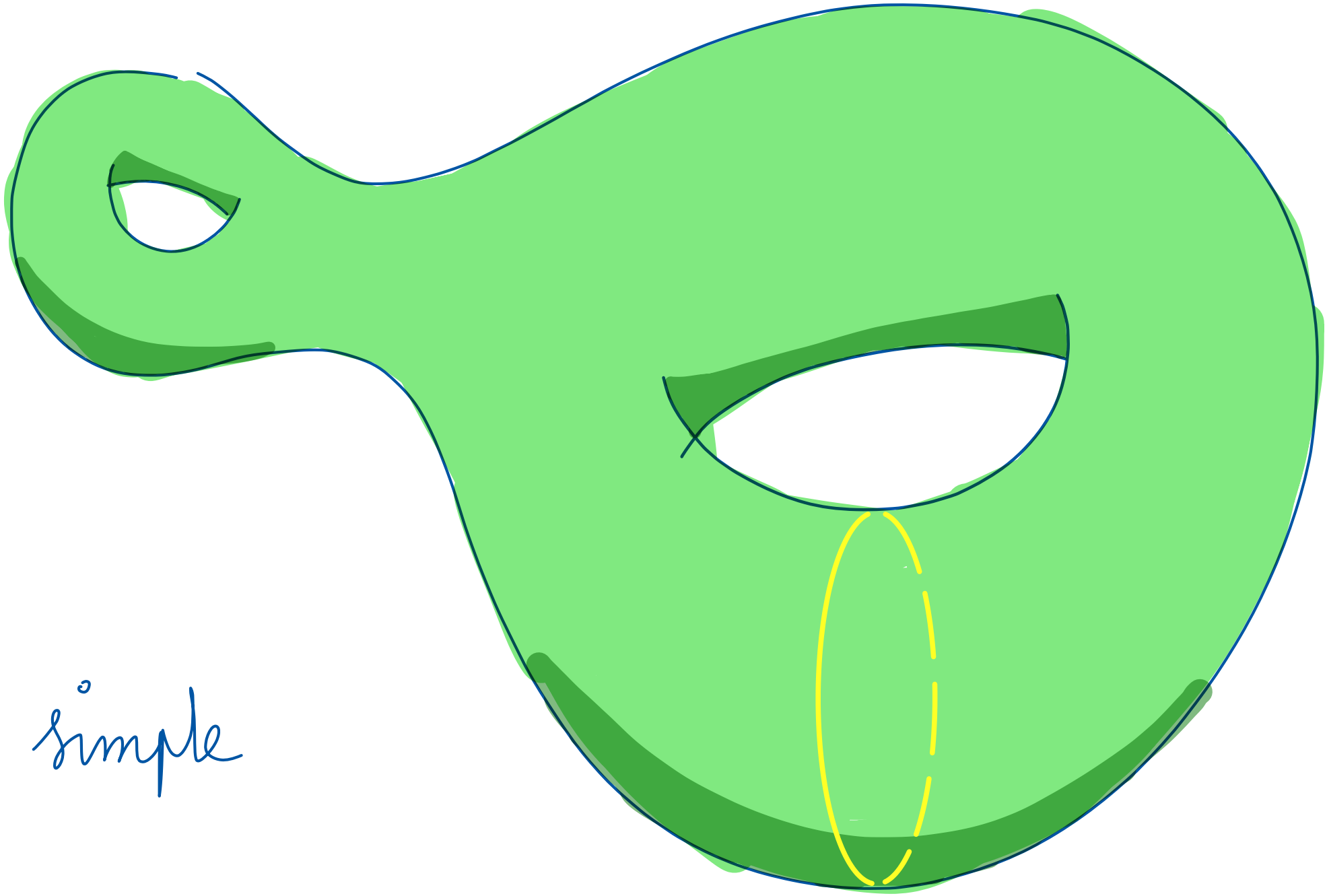
X hyperbolic surface of genus $g \geq 2$

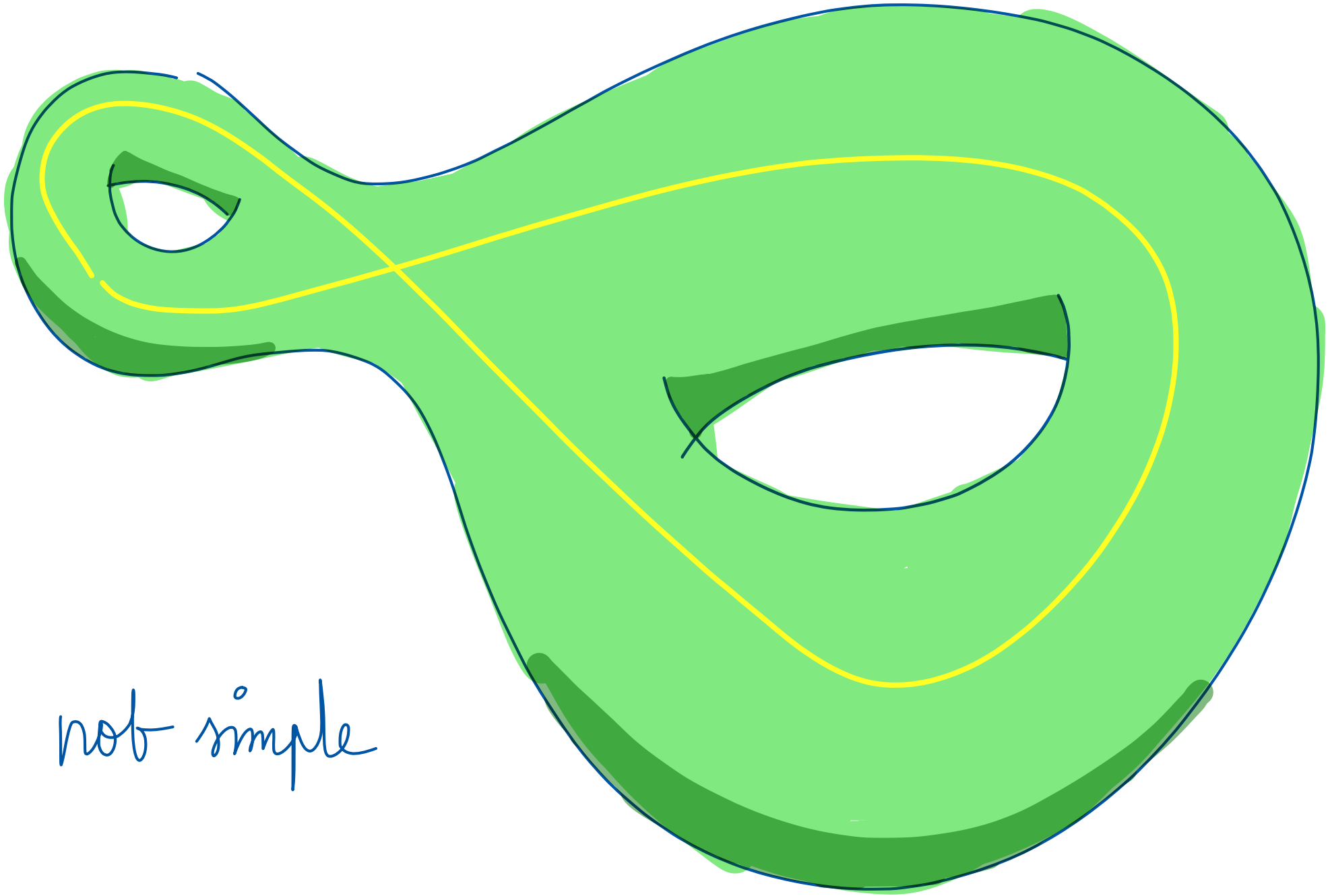
Definition

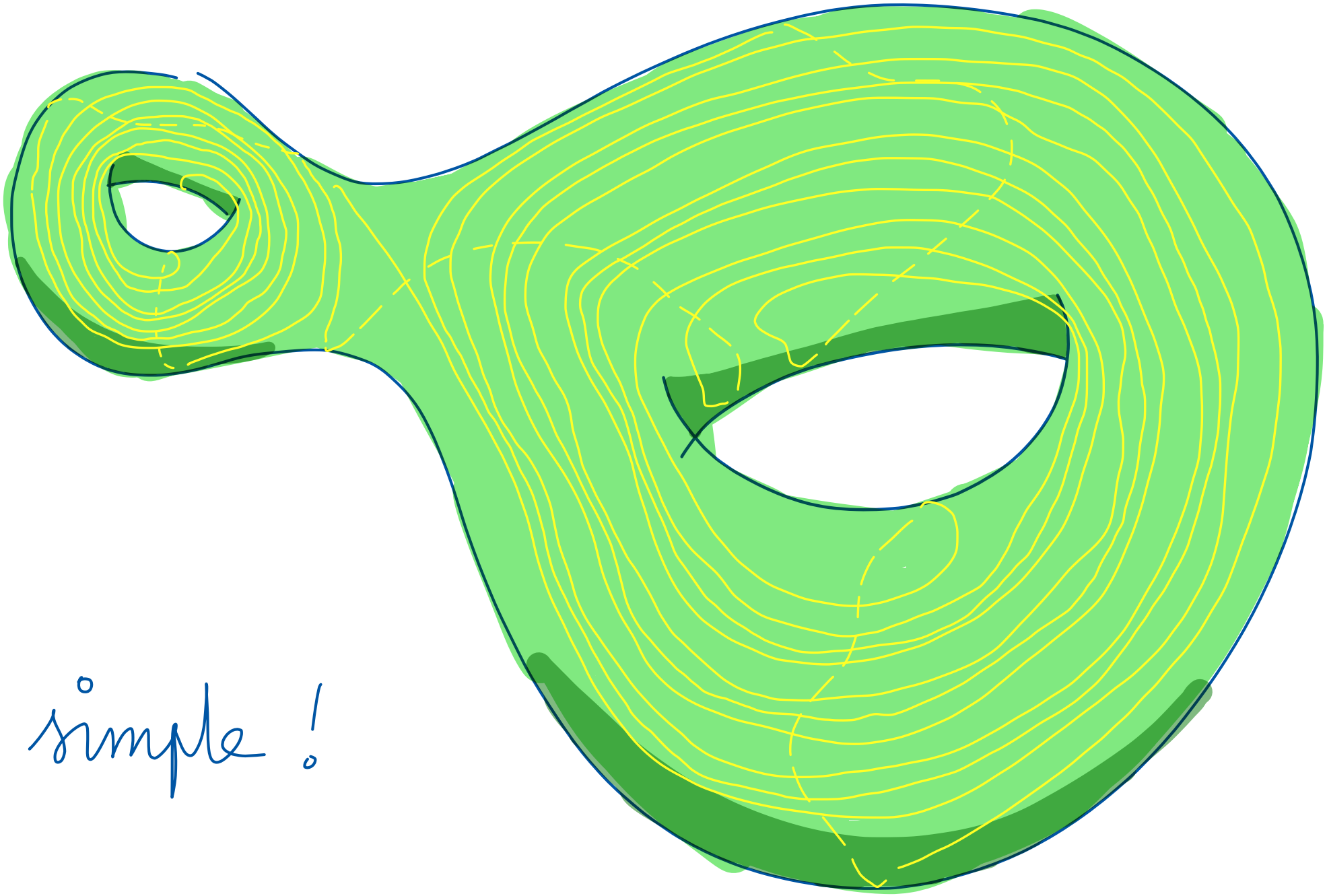
a closed geodesic is simple

if it does not interset itself



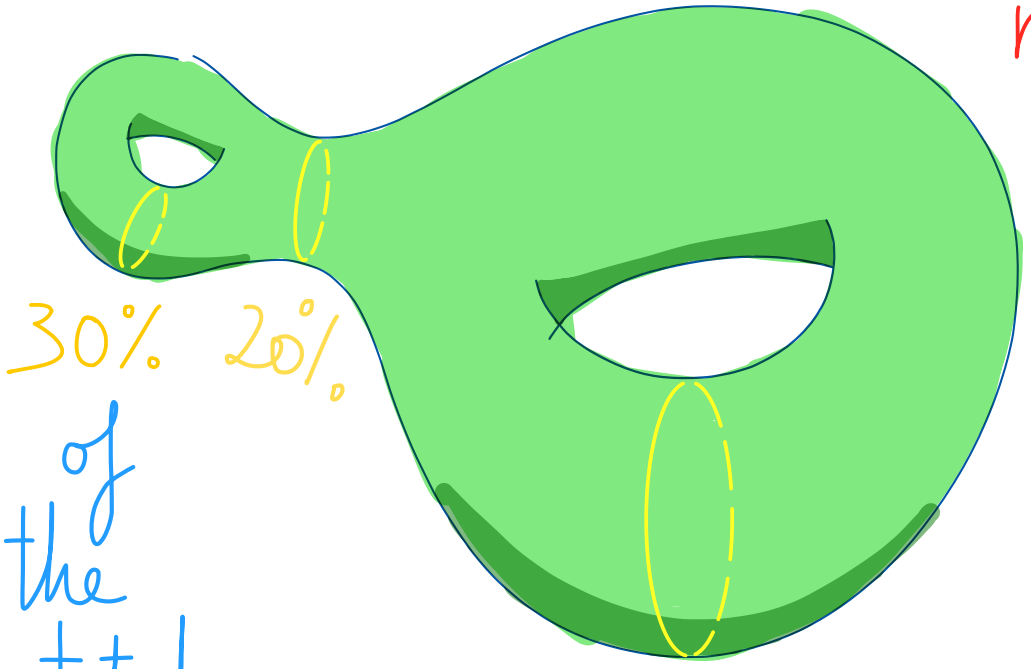

simple


not simple


simple !

A (primitive) multi-geodesic γ is a union of disjoint simple closed geodesics

Example



of the total length

normalized length vector

$\hat{\ell}_x(\gamma)$

$=$ in Example

$(50\%, 30\%, 20\%)$

Random multi-geodesics

$$S_{X,R} = \left\{ \alpha \text{ multi-geodesic on } X \;\middle|\; \ell_X(\alpha) \leq R \right\}$$

$\#\, S_X = \infty \Rightarrow$ no uniform proba measure

$\#\, S_{X,R} < \infty \Rightarrow$ equip $S_{X,R}$ with the uniform proba measure

length partition

$S_{X,R} \to$ a simplex

$\gamma \mapsto \hat{\ell}_{X,R}(\gamma)$

(50%, 30%, 20%)

Theorem (Delecroix - L)

- $\hat{\ell}_{X,R} \xrightarrow[R \to \infty]{(d)} L_g$

- $L_g \xrightarrow[g \to \infty]{(d)}$ Poisson-Dirichlet distribution of parameter $1/2$

- does **not** depend on X
- proof : dynamics on $\mathcal{M}_g$
- explicit : intersection theory on $\overline{\mathcal{M}}_{g,n}$

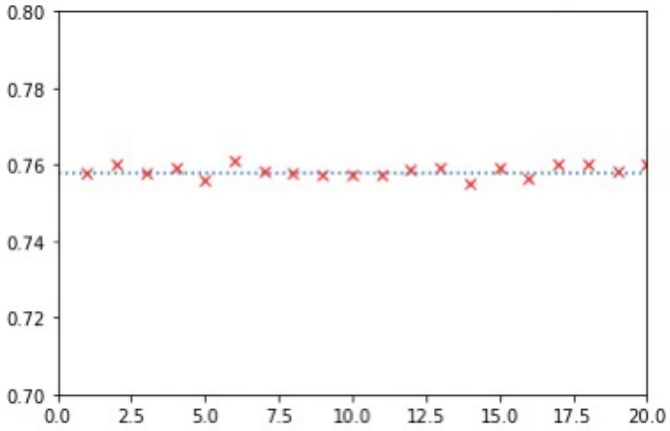
Corollary As $g \to \infty$, on average

longest component $\longrightarrow 75.8\%$

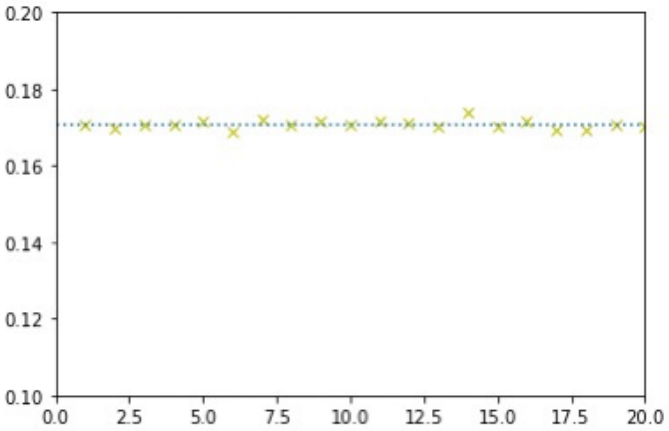
2^{nd} $\longrightarrow 17.1\%$

3^{rd} $\longrightarrow 4.9\%$

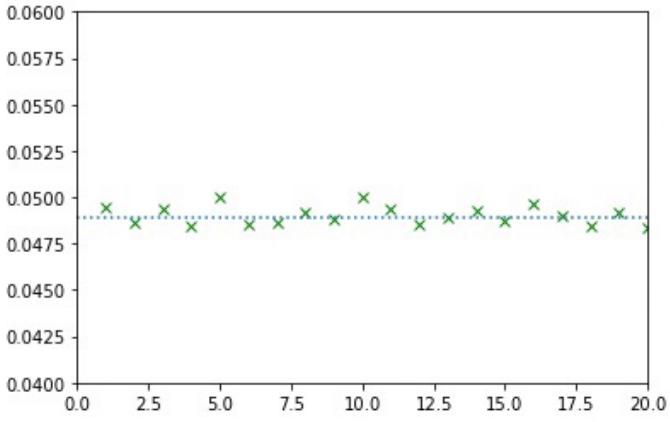
} 97.8%


0.80
0.78
0.76
0.74
0.72
0.70
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

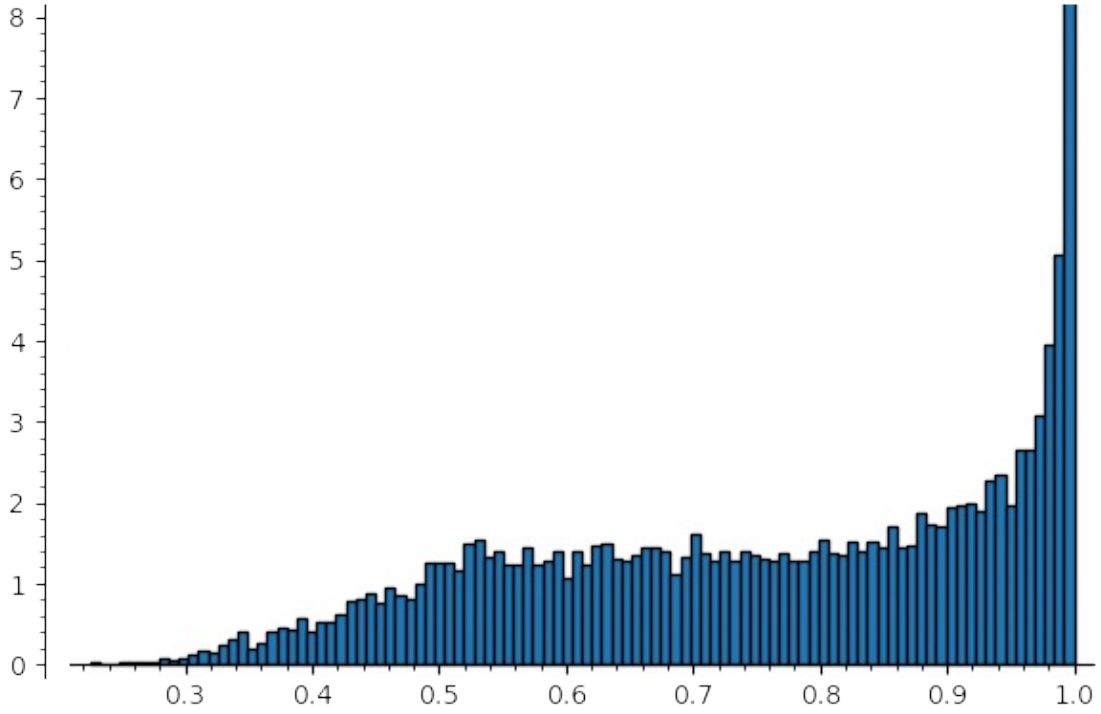
1st

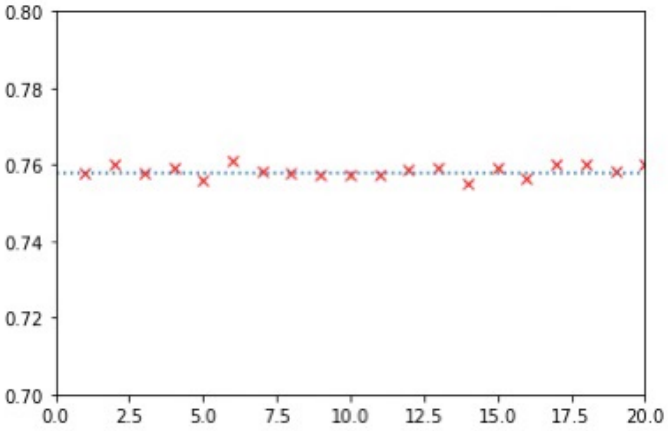

0.20
0.18
0.16
0.14
0.12
0.10
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

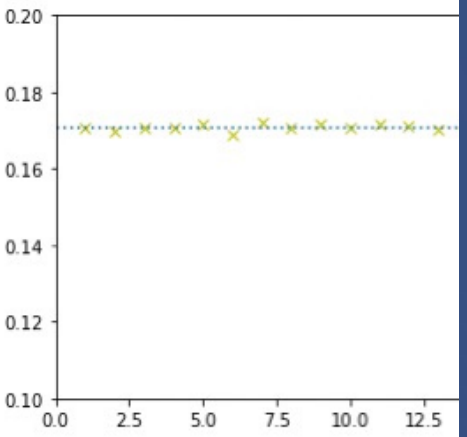
2nd


0.0600
0.0575
0.0550
0.0525
0.0500
0.0475
0.0450
0.0425
0.0400
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

3rd

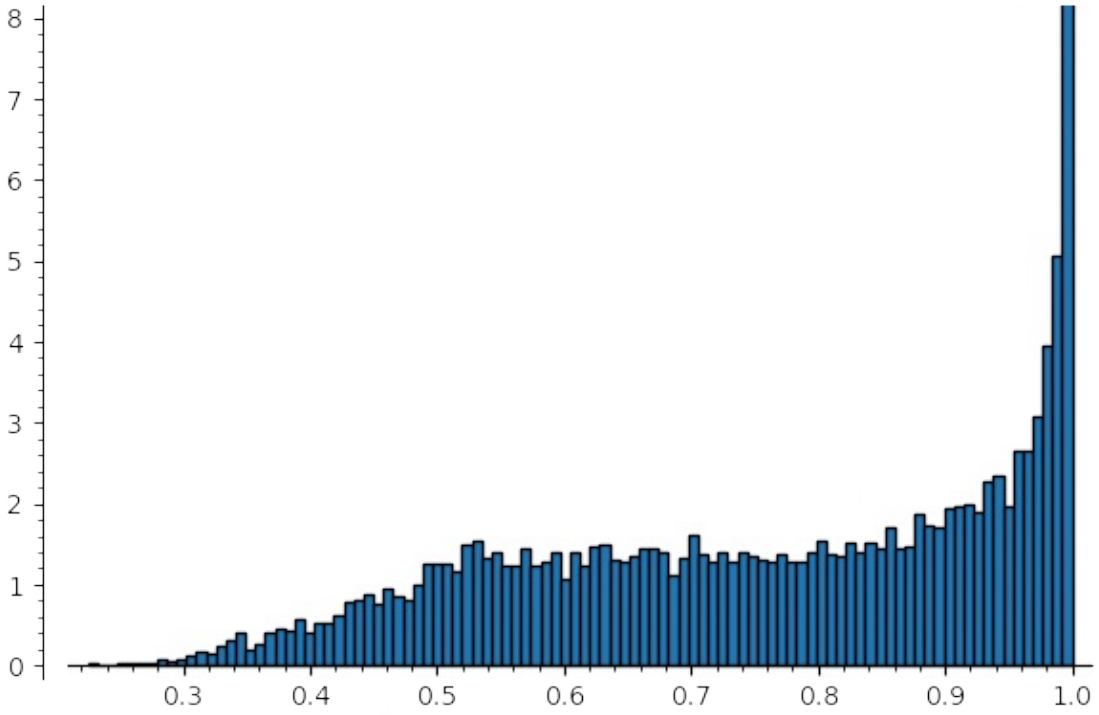

8
7
6
5
4
3
2
1
0
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

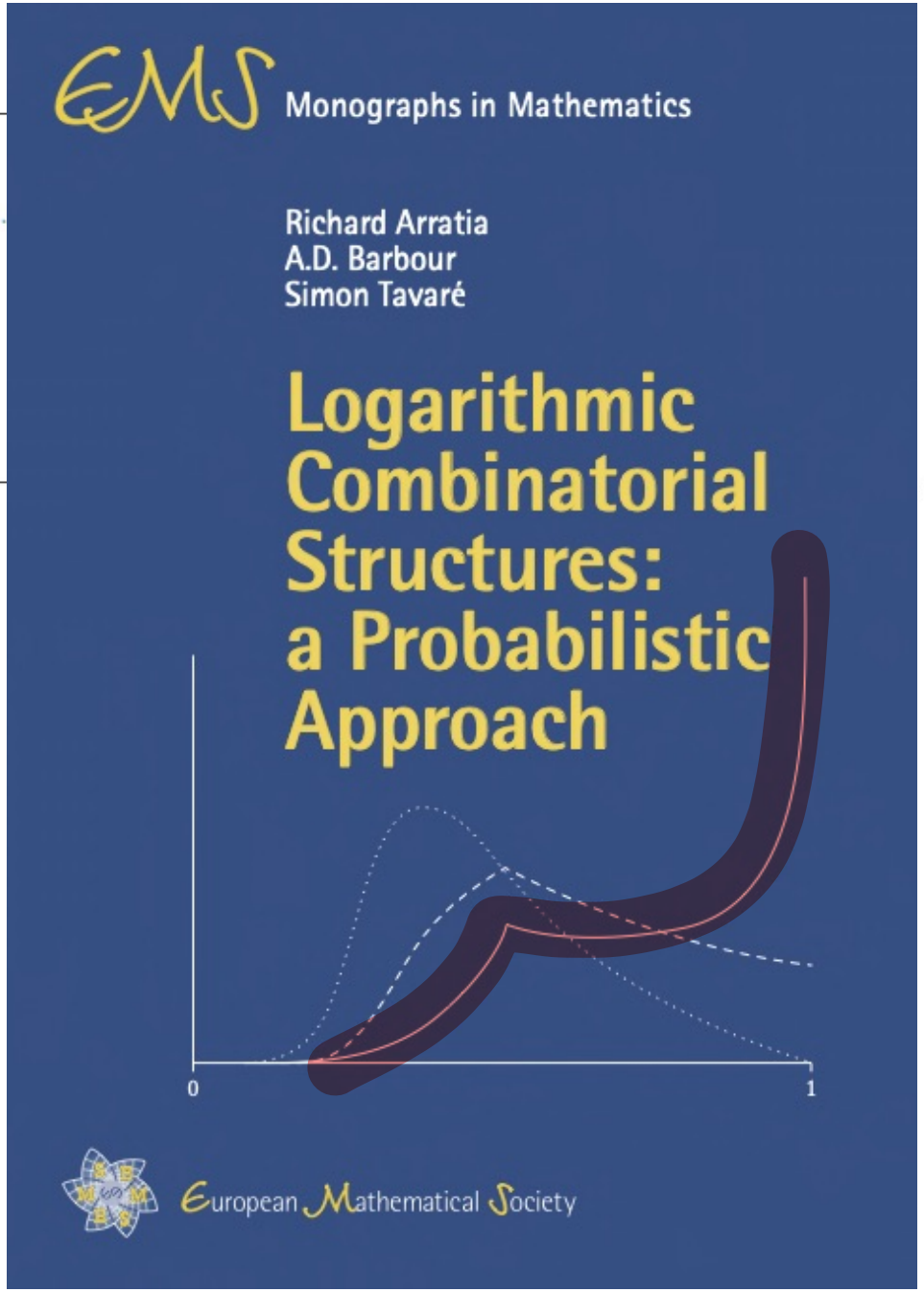

0.80
0.78
0.76
0.74
0.72
0.70
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0


0.20
0.18
0.16
0.14
0.12
0.10
0.0
2.5
5.0
7.5
10.0
12.5

1st

2nd


8
7
6
5
4
3
2
1
0
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0


EMS
Monographs in Mathematics
Richard Arratia
A.D. Barbour
Simon Tavaré
Logarithmic Combinatorial Structures: a Probabilistic Approach
0
1
European Mathematical Society

Thank you!

merci!