Geometry and spectra of hyperbolic surfaces

Bachelor thesis proposal

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> Nous qui sommes trace éphémère
> Dans la merveille et dans l'effroi
>
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The Laplacian is a ubiquitous differential operator in mathematics and physics, which can "hear" the shape of the space on which it lives. The aim of this project is to study the interplay between the geometry and the Laplacian spectrum of a hyperbolic surface (a 2-dimensional complete Riemannian manifold of constant negative curvature -1). The ultimate goal of this final year project is to understand the *Selberg trace formula*, which relates the closed geodesics and the eigenvalues of the Laplacian on a hyperbolic surface.

As a far-reaching noncommutative generalization of the Poisson summation formula, the Selberg trace formula bears a striking resemblance to the so-called explicit formulae in the analytic theory of L-functions, and evokes, the closed geodesics on a hyperbolic surface are somehow in analogy with prime numbers. Further generalizations of the Selberg trace formula have a crucial role to play in perhaps the most ambitious project in modern mathematics: the Langlands program.

We mostly follow [Ber]. Other references will be given later.

For further information, please contact: `mingkun.liu@uni.lu`

Prerequisites

Basic abstract algebra, Fourier and functional analysis.

References

[Ber] Nicolas Bergeron, *The Spectrum of Hyperbolic Surfaces*, 2016.