A glimpse into representation theory

Bachelor thesis proposal

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> But the essence of man is no abstraction inherent in each single individual. In reality, it is the ensemble of the social relations.
>
> *Theses on Feuerbach*, Karl Marx

Very roughly speaking, representation theory studies abstract algebraic structures by realizing them as symmetries of vector spaces.

Representation theory finds broad applications, ranging from number theory and combinatorics to geometry, probability theory, quantum mechanics, and quantum field theory; and also is one of the languages in which our deepest insights into Nature are expressed.

After studying some main general results of the theory, we shall focus on the representations of finite groups. If time permits, we will move forwards into more special (and more interesting) cases such as symmetric groups, Lie algebra, Lie groups, and quivers (or other fun stuff), depending on the taste of the student.

For those who have never seen representation theory, it is normal to feel lost and overwhelmed by its abstraction in the beginning. We hope that working through a considerable amount of examples will provide a significant aid towards overcoming the difficulties.

We mostly follow [EGHLSVY]. Other references will be given later.

For further information, please contact: `mingkun.liu@uni.lu`

Prerequisites

Linear algebra and basic abstract algebra.

References

[EGHLSVY] P. Etingof, O. Golberg, S. Hensel, T. Liu, A. Schwendner, D. Vaintrob, and E. Yudovina, *Introduction to representation theory*, 2011. https://math.mit.edu/~etingof/repb.pdf.