Luxembourg Workshop in Stochastic Analysis

February 10-11-12, 2025
University of Luxembourg

Conference programme

A conference organized by
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UNIVERSITÉ DU LUXEMBOURG

1. Programme

Monday
14:00 to 14:30 Welcome and Registration
14:30 to 15:15 Dario Trevisan
15:15 to 16:00 Charlotte Dion Blanc
Coffee break
16:30 to 17:15 Myra Shevchenko
17:15 to 18:00 Lucia Celli

Tuesday
9:00 – 9:45 Laurent Decreusefond
9:45 – 10:30 Florence Merlevède
Coffee break
11:00 – 11:45 Matthias Reitzner
11:45 – 12:30 Hermine Biermé
Lunch break
14:30 – 15:15 Sophie Langer
15:15 – 16:00 Alessia Caponera
Coffee break
16:30 – 17:15 Siva Athreya
17:15 – 18:00 Nikolai Leonenko
19:00–... Conference dinner

Wednesday
9:00 – 9:45 Yvik Swan
9:45 –10:30 Andreas Basse O Connor
Coffee break
11:00 – 11:45 Francesca Pistolato
11:45 – 12:30 Grégoire Szymanski

2. Abstracts

2.1. **Siva Athreya – International centre for Theoretical Sciences-TIFR**

Title: Interplay of vertex and edge dynamics for dense random graphs

Abstract: The large population limits of voter model in homogeneous populations, on lattices and on general fixed graphs are quite well understood. We consider a process where the graph itself is dynamic and changes in response to the voter model process, thus creating interaction between the two.

More precisely, we consider a dense random graph in which the vertices can hold opinion 0 or 1 and the edges can be closed or open. The vertices update their opinion at a rate proportional to the number of incident open edges, and do so by adopting the opinion of the vertex at the other end. The edges update their status at a constant rate, and do so by switching between closed and open with a probability that depends on their status and on whether the vertices at their ends are concordant or discordant. We understand the large n limit of this co-evolution and describe the limiting evolution.

This is joint work with Frank den Hollander and Adrian Roellin.

2.2. **Andreas Basse-O'Connor – Aarhus University**

Title: Quantitative bounds for high-dimensional non-linear functionals of Gaussian processes

Abstract: The Central Limit Theorem (CLT) is a fundamental result underlying various statistical tests and estimators, allowing for inferences about model parameters. In this talk, our main objective is to quantify the influence of dimensionality on the rate of convergence in the CLT. Specifically, we will show that the convergence rate exhibits sub-polynomial dimensional dependence in the hyper-rectangle metric for both short- and long-range dependent data. Our results are obtained from a combination of Stein's kernels and Malliavin Calculus.

The talk is based on joint work with David Kramer-Bang (Aarhus University).

2.3. **Hermine Biermé – Université de Tours**

Title: The anisotropy of 2D Gaussian random fields through their Lipschitz-Killing curvature densities

Abstract: I will present a joint work with Agnès Desolneux (CNRS, Centre Borelli, ENS Paris-Saclay) on the geometry of excursion sets of smooth stationary Gaussian fields. In this work we focus on the effect of anisotropy on their Lipschitz-Killing curvature densities (close from area, perimeter and Euler characteristics of excursion sets) and propose a new geometrical index of anisotropy, closely linked with isoperimetric inequalities.

2.4. **Alessia Caponera – LUISS Guido Carli**

Title: Multi-Scale CUSUM Tests for Time Dependent Spherical Random Fields

Abstract: In this talk we investigate the asymptotic behavior of structural break tests in the harmonic domain for time dependent spherical random fields. In particular, we prove a functional central limit theorem result for the fluctuations over time of the sample spherical harmonic coefficients, under the null of isotropy and stationarity; furthermore, we prove consistency of the corresponding CUSUM test, under a broad range of alternatives, including deterministic trend, abrupt change, and a nontrivial power alternative. Our results are then applied to NCEP data on global temperature: our estimates suggest that Climate Change does not simply affect global average temperatures, but also the nature of spatial fluctuations at different scales.

Based on a joint work with Anna Vidotto and Domenico Marinucci.

2.5. **Lucia Celli – University of Luxembourg**

Title: Entropic bounds for conditionally Gaussian vectors and applications to neural networks

Abstract: Using entropic inequalities from information theory, we provide new bounds on the total variation and 2-Wasserstein distances between a conditionally Gaussian law and a Gaussian law with invertible covariance matrix. We apply our results to quantify the speed of convergence to Gaussian of a randomly initialized fully connected neural network and its derivatives — evaluated in a finite number of inputs — when the initialization is Gaussian and the sizes of the inner layers diverge to infinity. Our results require mild assumptions on the activation function, and allow one to recover optimal rates of convergence in a variety of distances, thus improving and extending the findings of Basteri and Trevisan (2023), Favaro et al. (2023), Trevisan (2024) and Apollonio et al. (2024). One of our main tools are the quantitative cumulant estimates established in Hanin (2024).

The talk is based on a joint work with G. Peccati.

2.6. **Laurent Decreusefond – Telecom Paris**

Title: Analysis of stable distributions on convex cones

Abstract: Stable distributions on convex cones encompass the Gaussian law, usual stable distributions but also max-stable distributions, and many others. Inspired by the Mehler formula, for each stable distribution, we can use the definition of stability itself to construct a Markov semi-group whose stationary distribution is the chosen law. We apply this construction to max stable distributions and exhibit the rate of convergence in the coupon collector problem to the Gumbel distribution via Stein's method.

Stability also induces a Le Page series representation. We take advantage of the properties of the Poisson process to retrieve easily Poincaré and log-Sobolev inequalities for such distributions. We also give a new proof of the rate of convergence in the stable CLT for non integrable random variables.

2.7. **Charlotte Dion-Blanc – Sorbonne Université**

Title: Estimation for Hawkes-diffusion processes

Abstract: In this work, we study the process $(X_t, \lambda_t)_{t\geq 0}$, where X is a diffusion with jumps driven by a multivariate nonlinear Hawkes process, and λ is a piecewise deterministic Markov process (PDMP) that defines the stochastic intensity. We first explore the probabilistic properties of the process. Then, we focus on the nonparametric estimation of the coefficients in the stochastic differential equation (SDE) by minimizing various empirical contrast functions. Finally, we estimate the invariant density of the pair (X, λ) using kernel density estimation techniques. This step is crucial due to the potential applications of the model.

2.8. **Sophie Langer – University of Twente**

Title: On the statistical theory of deep learning

Abstract: A combination of algorithmic advances together with computational power and availability of data have led to a remarkable empirical success of deep learning across a range of domains. The effectiveness of the neural networks stands in stark contrast to the theoretical understanding of the procedure. Currently, we are far from a mature, unified theoretical theory of deep learning. Nevertheless, interesting results exist and one can roughly divide the existing literature in three parts, namely approximation, generalization and optimization. In this talk we mainly focus on generalization results. In particular, we contribute to the question why neural networks perform well on unknown new data. From a statistical learning perspective, one might answer this question by analyzing neural networks as estimators in a nonparametric regression setting. In recent results it was shown that neural networks are able to circumvent the so-called curse of dimensionality in case that suitable restrictions on the structure of the regression function hold. Under a general composition assumption on the regression function, one key feature of the neural networks used in these results is that their network architecture has a further constraint, namely the network sparsity. In this talk we show that we can get similar results also for least squares estimates based on simple fully connected neural networks with ReLU activation functions. Our analysis is twofold. In a first result, we show that fully connected networks derive a rate of convergence independent of the input dimension in case that the regression function fulfills some hierarchical composition structure. In a second step, the case, that the distribution of the predictor variable is concentrated on a manifold, is analyzed. As an outlook of this talk, we present finally a result which analyzes all three aspects of deep learning, namely approximation, generalization and optimization, simultaneously. In a simplified setting, i.e., for neural networks with one hidden layer, we show that neural networks learned by gradient descent circumvent the curse of dimensionality in case that the Fourier transform of the regression functions decays suitable fast.

2.9. **Nikolai Leonenko - Cardiff University**

Title: Asymptotic Properties of Estimators of Entropy-type Functionals via Nearest Neighbour Graphs

Abstract: The varentropy is a measure of the variability of the information content of random vector and it is invariant under affine transformations. In the first part of the talk we introduce the statistical estimate of varentropy of random vector based on the nearest neighbour graphs (distances). The asymptotic unbiasedness and consistency of the estimates are established [5]. The fist part of the talk is based on joint results with Yu Sun (LSE, UK) and Emanuele Taufer (University of Trento, Italy).

In the second part of the talk we consider similar problems for stationary m- dependent multivariate processes with m-dependence and their applications for estimation Shannon and Renyi entropies [2] and varentropy. The second part of the talk is based on joint results with Vitalii Makogin (Ulm University, Germany). The entropy based goodness of fit tests are also discussed in the spirit of the results of the papers [1,3].

Reference

- [1] Cadirci, M. S; Evans, D.; Leonenko, N.; Makogin, V. (2022); Entropy-based test for generalised Gaussian distributions. Computational Statistics and Data Analysis. 173 , Paper No. 107502
- [2] Leonenko, N., Makogin, V. (2025); Limit theorems for point processes. Manuscript
- [3] Leonenko, N., Makogin, V. and Cadirci, M.S. (2021); The entropy based goodness of fit tests for generalized von Mises-Fisher distributions and beyond, Electronic Journal of Statistics, 15, N2, 6344-638
- [4] Leonenko N, Pronzato, L. and Savani, V., (2008), A class of Renyi information estimators for multidimensional densities, 36, N5, Annals of Statistics, 2153-2182, 36,N5 Corrections, Annals of Statistics , 2010, 38, N6, 3837-3838
- [5] Leonenko, N., Sun, Y., and Taufer, E. (2024). Varentropy estimation via nearest neighbour graphs. ArXiv, 2402.09374, p.1-51

2.10. **Florence Merlevede – Université Gustave Eiffel**

Title: On the weak invariance principle for stationary random fields under $\mathbb{L}^1$-projective conditions when commuting filtrations are used

Abstract: We consider a field $f \circ T_1^{i_1} \circ \cdots \circ T_d^{i_d}$ where $T_1, \ldots, T_d$ are ergodic and completely commuting transformations in the sense of Gordin. In this talk we are interested by giving sufficient $\mathbb{L}^1$-projective conditions ensuring not only that the normalized partial sums indexed by quadrants converge in distribution to a normal random variable but also that the functional form of this result holds. For the central limit theorem, the proof combines a truncated orthomartingale approximation with a recent CLT for martingale random fields due to Volný. For the functional form, a new maximal inequality is needed and is obtained via truncation techniques, blocking arguments and orthomartingale approximations. Application to bounded Lipschitz functions of linear fields whose innovations have moments of a logarithmic order will be provided as well as application to completely commuting endomorphisms of the m-torus. In the latter case, the conditions can be expressed in terms of the $\mathbb{L}^1$-modulus of continuity of f.

This is a joint work with J. Dedecker and C. Cuny.

2.11. **Francesca Pistolato – University of Luxembourg**

Title: Limit theorems for p-domain functionals of stationary Gaussian random fields

Abstract: Integral functionals of stationary Gaussian random fields provide a unified framework to describe, for instance, the qth variation of stochastic processes, geometric functionals such as the excursion volume, and even sums of random variables. The topic of the talk is the *p-domain functionals* of stationary Gaussian random fields, a particular class of integral functionals where the domain of integration is a product of lower-dimensional domains, and their asymptotic behavior as the size of the integration domain grows.

A cornerstone in the understanding of such functionals is Breuer-Major theorem, which links the *short-range dependence* of the integrand field, that is, the integrability of its covariance function, together with the standardized functional satisfying a Central Limit Theorem (CLT).

In this talk, under further assumptions on the covariance function, we will show how the asymptotic behavior of p-domain functionals can be simply obtained from that of 1-domain functionals, explaining in a new light and in a more systematic way some results from the recent literature. For instance, the result provides a new proof of the convergence of the qth variation of the fractional Wiener sheet to a Gaussian distribution, proved by A. Réveillac, M. Stauch and C. Tudor in 2012, and improves their estimate on the rate of convergence. Moreover, it allows us to produce new examples of long-range dependent fields satisfying CLTs.

The talk is based on a joint work with N. Leonenko, L. Maini and I. Nourdin.

2.12. **Matthias Reitzner – Universität Osnabrück**

Title: Poisson-Delaunay approximation

Abstract: For a Borel set A and a stationary Poisson point process of uniform intensity, the Poisson-Delaunay approximation of A is the union of all Delaunay cells generated by the points of the point process with center in A. It is shown that the volume of the Poisson-Delaunay approximation is an unbiased estimator for the volume of A, variance bounds and a quantitative central limit theorem using Steins method are given. Also the asymptotic behavior of the volume of the symmetric difference between A and the Poisson-Delaunay approximation is investigated.

2.13. **Radomyra Shevchenko – Université Côte d'Azur**

Title: Drift parameter estimation for a fractional interacting particle system

Abstract: We consider a system of interacting particles with Lipschitz continuous drift functions, driven by additive fractional Brownian motions with $H \in [1/2, 1)$. For this system, we address the drift parameter estimation problem from continuous observations over a fixed time interval, assuming that the drift depends linearly on an unknown parameter vector. We propose estimators inspired by the least squares approach, demonstrate their consistency and asymptotic normality as the number of particles tends to infinity, and present a numerical study illustrating our findings. The proofs rely on establishing a quantitative propagation of chaos result for the Malliavin derivatives of the system.

This talk is based on joint work with Chiara Amorino and Ivan Nourdin.

2.14. **Yvik Swan – Université Libre de Bruxelles**

Title: Some Stein operators for multivariate distributions and applications

Abstract: The title of this talk is basically its abstract : we present some Stein operators for multivariate distributions, along with their applications. The first application concerns the very classical problem of multivariate normal approximation; the second application concerns parameter estimation for Fisher-Bingham distributions on the sphere.

2.15. **Gregoire Szymanski – University of Luxembourg**

Title: Robust inference for rough volatility

Abstract: Rough volatility models have emerged as a powerful framework to capture the intricate dynamics and irregularities of financial markets. These models, characterized by fractional Brownian motion with a Hurst parameter $H < 1/2$, provide an effective description of the high-frequency, rough behavior of stochastic volatility. In this presentation, we offer a comprehensive overview of three distinct contributions that tackle various facets of the problem of estimating the Hurst parameter H. We discuss its implications from a financial perspective and address the statistical limitations inherent to this approach. We focus on the estimation of H from discrete price observations within a semi-parametric setting, without assuming any predefined relationship between volatility estimators and true volatility. The proposed estimation methodology is robust to market complexities, including the presence of jumps and microstructure noise, ensuring its applicability in realistic financial settings.

2.16. **Dario Trevisan – University of Pisa**

Title: Wide Deep Neural Networks with Gaussian Weights are Very Close to Gaussian Processes

Abstract: We establish novel rates for the Gaussian approximation of random deep neural networks with Gaussian parameters (weights and biases) and Lipschitz activation functions, in the wide limit. Our bounds apply for the joint output of a network evaluated any finite input set, provided a certain non-degeneracy condition of the infinite-width covariances holds. We demonstrate that the distance between the network output and the corresponding Gaussian approximation scales inversely with the width of the network, exhibiting faster convergence than the naive heuristic suggested by the central limit theorem. We also apply our bounds to obtain theoretical approximations for the exact Bayesian posterior distribution of the network, when the likelihood is a bounded Lipschitz function of the network output evaluated on a (finite) training set. This includes popular cases such as the Gaussian likelihood, i.e. exponential of minus the mean squared error. Talk based on arXiv:2312.11737.