RMATH | MATHEMATICS RESEARCH UNIT

UNIVERSITÉ DU LUXEMBOURG

Fonds National de la Recherche Luxembourg

Workshop on Gauge Theory and Noncommutative Geometry

June 19-21, 2012, University of Luxembourg

Venue: Room B02 (main building)

Programme:

Tuesday, July 19

14:00-14:30	Welcome / Registration
14:30-15:30	*T. Brzeziński*
15:30-16:00	Coffee break
16:00-17:00	*P. M. Hajac*

Wednesday, June 20

10:00-11:00	*S. Majid*
11:00-11:30	Coffee break
11:30-12:30	*R. Szabo*
12:30-14:30	Lunch break
14:30-15:30	*T. Masson*
15:30-16:00	Coffee break
16:00-17:00	*W. van Suijlekom*
20:00	Conference dinner

Thursday, June 21

10:00-11:00	*A. Schenkel*
11:00-11:30	Coffee break
11:30-12:30	*G. Landi*

Titles and Abstract:

- **Tomasz Brzeziński**: *'Bundles over quantum weighted projective spaces'*
  Abstract. Quantum weighted projective spaces are defined as quotients of weighted circle actions on quantum spheres and other quantum spaces. They are examples of deformations of orbifolds. We describe in detail algebraic structure of the lower dimensional cases: quantum weighted projective complex lines and quantum weighted projective real planes. In addition we construct principal $U(1)$-bundles and associated line bundles over these spaces. Joint work with Simon A Fairfax.

- **Piotr M. Hajac:** *'Principal coactions'*
  Abstract. On 19 December 1949, Henri Cartan introduced principal actions of topological groups on topological spaces as actions that are free and proper. Thus he defined the fundamental concept of a principal bundle, nowadays a standard tool of differential geometry or gauge theory. In this lecture, we rephrase Cartan's definition in terms of C^*-algebras, and study its meaning for general noncommutative C^*-algebras. In particular, we focus attention on the unital C^*-algebras describing compact quantum principal bundles. We show that free actions of compact quantum groups on unital C^*-algebras always correspond to principal coactions of their Hopf algebras on the Peter-Weyl subalgebras of the algebraic direct sums of isotypical components. This puts us in the category of principal comodule algebras. We show advantages of working in this category by reviewing some major theorems concerning principal comodule algebras or more general principal coactions of coalgebras on algebras.

- **Giovanni Landi:** *'Gauge Fields and q-Vortices over the Quantum Sphere'*
  Abstract. We extend equivariant dimensional reduction techniques to spaces which are the product of a Kaehler manifold M with a noncommutative space. In particular, we work out the reduction of bundles which are equivariant under the natural action of the quantum $SU(2)$ group, and also of invariant gauge connections on these bundles. The reduction of Yang–Mills gauge theory from the product space leads to deformed quiver gauge theories on M. Corresponding vacuum moduli spaces are generally better behaved than their undeformed counterparts, but much more constrained by the q-deformation. (Based on work with R. Szabo).

- **Shahn Majid:** *'Noncommutative Black Holes'*
  Abstract. Let M be a Riemannian manifold admitting a conformal Killing vector. We regard the cross product algebra by the action of the vector field as a quantization of $M \times \mathbb{R}$ and construct a family of noncommutative differential calculi on it. In the process we construct a noncommutative analogue of the wave operator for any static metric on $M \times \mathbb{R}$ with the original metric on M as spatial part. This class of metrics includes the Schwarzschild black hole and we describe first predictions

of the resulting noncommutative wave operator in a certain approximation. Effects include making finite the classically infinite time dilation at the black hole event horizon.

- **Thierry Masson:** *'Gauge Theories on Transitive Lie Algebroids Inspired by Noncommutative Gauge Theories'*
  Abstract. Inspired by noncommutative methods, transitive Lie algebroids can be used as a framework for gauge fields theories. I will present the necessary tools to write down gauge theory action functionals: forms, integration, Hodge star operators... These gauge field theories are of Yang-Mills-Higgs type, and I will show how ordinary Yang-Mills theories are included in these theories. Comparisons with some noncommutative gauge field theories will be given.

- **Alexander Schenkel:** *'Product module homomorphisms and connections in twist deformed NC geometry'*
  Abstract. I will report on a joint work with Paolo Aschieri. Let H be a Hopf algebra, A an H-module algebra and V an H-module A-bimodule. We study the behavior of the right A-linear endomorphisms of V under twist deformation. We in particular construct a bijective quantization map to the right $A_\star$-linear endomorphisms of $V_\star$, with $A_\star, V_\star$ denoting the usual twist deformations of A, V. The quantization map is extended to right A-linear homomorphisms between H-module A-bimodules and to right connections on V. We then investigate the tensor product of linear maps between H-modules. Given a quasitriangular Hopf algebra we can define an H-covariant tensor product of linear maps, which restricts for H-module A-bimodules to a well-defined tensor product of right A-linear homomorphisms on tensor product modules over A. This also requires a quasi-commutativity condition on the algebra and bimodules. Using this tensor product we can construct a new lifting prescription of connections to tensor product modules, generalizing the usual prescription to also include nonequivariant connections.

- **Richard J. Szabo:** *'Quantization of 2-plectic Manifolds'*
  Abstract. Recent developments in string theory and M-theory motivate the need for developing notions of quantum geometry involving suitable higher structures, such as 3-algebras. In this talk we consider one such higher structure which is provided by the categorification of a symplectic manifold, called a 2-plectic manifold. Geometric quantization in this higher setting requires replacing prequantum line bundles with prequantum gerbes. We offer a perspective on such a higher quantization which involves transgressing gerbes to loop space, where the 2-plectic structure becomes an ordinary symplectic structure. We demonstrate that a tractable quantization scheme in the setting of loop spaces is provided by the geometric quantization of a suitable integrating symplectic groupoid for the loop space Poisson structure.

- **Walter D. van Suijlekom:** *'Moduli Spaces of Instantons on Toric Noncommutative Manifolds'*

  Abstract. We present some recent results on moduli spaces of $U(2)$ instantons on toric noncommutative 4-manifolds. After studying the analytic aspects of $U(n)$ gauge theory on toric noncommutative manifolds, we show that each such moduli space is either empty or a smooth Hausdorff manifold whose dimension we explicitly compute. In the special case of the toric deformation of the four-sphere, we find that the moduli space of $U(2)$ instantons with fixed Chern number k is a smooth manifold of dimension $8k-3$. (joint work with Simon Brain and Giovanni Landi)