MONODROMY OF SUBREPRESENTATIONS AND IRREDUCIBILITY OF LOW DEGREE AUTOMORPHIC GALOIS REPRESENTATIONS

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ABSTRACT. Given a compatible system $\{\rho_{\lambda} : \operatorname{Gal}_{K} \to \operatorname{GL}_{n}(E_{\lambda})\}_{\lambda}$ of semisimple λ -adic representations of a number field K satisfying mild local conditions, we prove that for almost all λ any type A irreducible subrepresentation of $\rho_{\lambda} \otimes \overline{\mathbb{Q}}_{\ell}$ is residually irreducible. We apply this result and some potential automorphy theorem to prove that $\rho_{\lambda} \otimes \overline{\mathbb{Q}}_{\ell}$ is residually irreducible for almost all λ if the compatible system is attached to a regular algebraic, polarized, cuspidal automorphic representation of $\operatorname{GL}_{n}(\mathbb{A}_{\mathbb{Q}})$ and $n \leq 6$.