READING SEMINAR ON ALGEBRAIC GROUPS

Organizers: Bryan Advocaat, Andrea Conti

The seminar will take place Tuesdays 10:30-12:00 in Room MNO 1.010. The talks will be given in hybrid mode. If you want to volunteer for a talk please let us know.

Our main goal will be to explain the classification of reductive groups in terms of their root systems. We will mainly be using the book *Linear Algebraic Groups* by A. Borel. Other useful references are the books of the same title by Humphreys and Springer. We took inspiration from the program of a reading group by Moritz Kerz (https://kerz.app.uni-regensburg.de/archive/ss12/oberseminar.pdf).

talk 1: Overview about algebraic groups, some basic ideas, why they are useful. Basics of the algebraic geometry we will need. (Andrea)

talk 2: Sections 1 and 2 of Chapter I. The basic definitions/propositions regarding Algebraic groups and examples.

talk 3: Sections 3 and 4 of Chapter I. This includes the definition of the Lie algebra of an algebraic group, some properties and examples.

talk 4: Construct quotients of algebraic groups, see Theorem 6.8 in Chapter 2, Section 6. Cover Section 7 as well. Most proofs can be skipped.

talk 5: Chapter 3, Section 8 on tori. Basic definitions, weights and roots.

talk 6: Centralizers and conjugacy classes of semi-simple elements, Lie-Kolchin Theorem (Corollary 10.5). Section 9 and part of 10.

talk 7: Structure theorem of connected solvable groups (Thm. 10.6), one-dimensional groups are isomorphic to $\mathbf{G}_a$ or $\mathbf{G}_m$ (Thm. 10.9). Definition of Borel subgroups and conjugacy of Borel subgroups (Thm. 11.1).

talk 8: Borel subgroups, Cartan subgroups, maximal tori, theorem of Chevalley (Thm.11.16), Weyl group, radical, reductive and semi-simple groups. Section 11.

talk 9: Further properties of Cartan subgroup, regular and singular tori, groups of semisimple rank one, roots of a reductive group (Thm. 13.18). Sections 12 and 13.

talk 10: Explain abstract root systems and show how a reductive group (over an algebraically closed field) gives rise to a root system. Sections 14.1-14.8.

talk 11: Bruhat decomposition (Section 14.12), parabolic subgroups, Levi subgroups. Decomposition of a semi-simple group into simple factors, Bruhat decomposition, parabolic subgroups, Levi subgroups. Sections 14.9 - 14.26.