Proposed schedule for the seminar "Adic spaces"
University of Luxembourg, Winter Semester 2020

The seminar will take place on Tuesdays from 10:30 to 12:30 in room MSA 3.220, except for September 29th (room 3.100) and November 3 (room 3.190).

The main reference for the seminar is the script by Wedhorn [Wed12]. Additional suggested references are:
- Morel's script [Mor19];
- Weinstein's script from the Arizona Winter School 2017 [Wei17], and the videos of his lectures;
- Conrad's brief introduction [Con18];
- The brief introduction in Scholze's perfectoid paper [Sch12, pages 2–8].

One can also have a look at the exercises from the course by Venjakob and Ludwig in Heidelberg [Lud20] and those by Mieda for the Arizona Winter School [Mie17].

The foundations of the theory go back to Huber and most of the material can also be found in his original papers and book [Hub93; Hub94; Hub96].

1. Introduction: [Andrea, 29/9] *References:* [Con18], [Wei17], [Sch12].
Recall the motivation for introducing rigid spaces as nonarchimedean analogues of complex analytic spaces, and the basic constructions from the previous seminar: rigid spaces as locally ringed spaces, locally isomorphic to affinoid algebras, and equipped with the Tate G-topology. Explain the shortcomings of this theory and how adic spaces should solve them. If time allows mention formal schemes and how these are also generalized by adic spaces.

2. Valuations [?, 6/10] *References:* [Wed12, Sections 1 and 2].
Introduce valuations in the sense of Huber (what one would normally call seminorms), of arbitrary rank. Prove some basic properties of valuation rings. This part may be a bit long but it is just basic algebra and one could leave out some of the less interesting proofs.

3. Spectral and sober spaces [?, 13/10] *References:* [Wed12, Section 3], [Con18].
Define spectral spaces and sober spaces and prove their basic properties. One could read in [Con18] and explain why the notion of sober space is important for what follows.

4. Valuation spectra [, 20/10] *References:* [Wed12, Section 4].
Define the valuation spectrum of an arbitrary ring; it is a topological space. Discuss generizations and specializations in a valuation spectrum.

5. Non-archimedean rings [?, 27/10] *References:* [Wed12, Section 5].
Define non-archimedean topological rings, their subrings of power-bounded and of topologically nilpotent elements, and make examples.

6. f-adic rings and Tate rings [?, 3/11] *References:* [Wed12, Section 6].

7. Adic spectra of affinoid rings, I [?, 10/11] *References:* [Wed12, Subsections 7.1–7.4].
Define affinoid rings and their adic spectra, that are topological spaces. These are not yet affinoid adic spaces.

8. Adic spectra of affinoid rings, II [?, 17/11] *References:* [Wed12, Subsections 7.5–7.7].
Define the set of analytic points of the adic spectrum of a ring. Prove some basic properties of adic spectra and make examples.

9. Adic spaces, I [?, 24/11] *References:* [Wed12, Subsections 8.1–8.2].

Define a structure presheaf on the adic spectrum of an affinoid ring. Define affinoid adic spaces: they are adic spectra of affinoid rings for which the structure presheaf is a sheaf. Use them as building blocks to define adic spaces.

10. Adic spaces, II [?, 1/11] *References:* [Wed12, Subsections 8.1–8.2].

Discuss analytic points, morphisms, and fiber products of adic spaces.

11. From rigid analytic spaces and formal schemes to adic spaces [?, 8/12] *References:* [Wed12, Sections 9-10], [Hub94, Section 4].

Show how to attach an adic space to a rigid analytic space. Without the details, explain what a formal scheme is and how to attach a rigid analytic generic fiber and an adic space to a locally Noetherian formal scheme. Explain how all of these constructions are compatible.

If necessary we could have a talk in the last week before Christmas, on a different day if Tuesday is the Number Theory Day. One could either finish what is left of the above program, or give (if manageable) a very quick idea of some applications. Typical applications are: defining perfectoid spaces [Sch12], constructing the spectral halo of the eigencurve [AIP15], constructing the fundamental curve of p-adic Hodge theory [FF18].

References

[AIP15] F. Andreatta, A. Iovita, and V. Pilloni. "Le halo spectral". In: (2015), preprint.

[Con18] B. Conrad. *A brief introduction to adic spaces.* 2018. URL: `http://math.stanford.edu/~conrad/papers/Adicnotes.pdf`.

[FF18] L. Fargues and J.-M. Fontaine. *Courbes et fibrés vectoriels en théorie de Hodge p-adique.* Vol. 406. Astérisque. Paris: Soc. Math. France, 2018, pp. xiii+382.

[Hub93] R. Huber. "Continuous valuations". In: *Math. Z.* 212.3 (1993), pp. 455–477. ISSN: 0025-5874. DOI: `10.1007/BF02571668`. URL: `https://doi.org/10.1007/BF02571668`.

[Hub94] R. Huber. "A generalization of formal schemes and rigid analytic varieties". In: *Math. Z.* 217.4 (1994), pp. 513–551. ISSN: 0025-5874. DOI: `10.1007/BF02571959`. URL: `https://doi.org/10.1007/BF02571959`.

[Hub96] Roland Huber. *Étale cohomology of rigid analytic varieties and adic spaces.* Aspects of Mathematics, E30. Friedr. Vieweg & Sohn, Braunschweig, 1996, pp. x+450. ISBN: 3-528-06794-2. DOI: `10.1007/978-3-663-09991-8`. URL: `https://doi.org/10.1007/978-3-663-09991-8`.

[Lud20] J. Ludwig. *Exercises on adic spaces.* 2020. URL: `https://typo.iwr.uni-heidelberg.de/fileadmin/groups/arithgeo/templates/data/Judith_Ludwig/home/adic_exercises.pdf`.

[Mie17] Yoichi Mieda. *Arizona Winter School 2017: Problems on adic spaces and perfectoid spaces.* 2017. URL: `swc.math.arizona.edu/aws/2017/2017MiedaProblems.pdf`.

[Mor19] S. Morel. *Adic Spaces.* 2019. URL: `https://web.math.princeton.edu/~smorel/adic_notes.pdf`.

[Sch12] Peter Scholze. "Perfectoid spaces". In: *Publ. Math. Inst. Hautes Études Sci.* 116 (2012), pp. 245–313. ISSN: 0073-8301. DOI: `10.1007/s10240-012-0042-x`. URL: `https://doi.org/10.1007/s10240-012-0042-x`.

[Wed12] T. Wedhorn. *Adic Spaces.* 2012. URL: `https://arxiv.org/pdf/1910.05934.pdf`.

[Wei17] J. Weinstein. *Arizona Winter School 2017: Adic Spaces.* 2017. URL: `http://swc.math.arizona.edu/aws/2017/2017WeinsteinNotes.pdf`.