SELBERG'S THEOREM VIA STEIN'S METHOD

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We consider a random variable U_T, distributed uniformly over the set $[0,T]$. The goal of this talk is to use Stein's method techniques to give a proof of the celebrated Selberg's theorem, which asserts that the random variable

$$\mathbf{X}_T = \frac{1}{\sqrt{\log\log(T)}} \log(\zeta(\frac{1}{2} + \mathbf{i}U_T))$$

converges in distribution to a standard complex Gaussian random variable; namely, a random variable of the form $\mathbf{N} = (N_1, N_2)$, where N_1 and N_2 are independent centered Gaussian variables with variance $\frac{1}{2}$. As a byproduct of our analysis, we will prove sharp (non-uniform) bounds for

$$|\mathbb{E}[f(\mathbf{X}_T)] - \mathbb{E}[f(\mathbf{N})]|,$$

for any polynomial $f : \mathbb{R}^2 \to \mathbb{R}$.

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