A recurrence relation for elliptic divisibility sequences

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In literature, there are two different definitions of elliptic divisibility sequences. The first one says that a sequence of integers $\{h_n\}_{n\in\mathbb{N}}$ is an elliptic divisibility sequence if it verifies the recurrence relation $h_{m+n}h_{m-n}h_r^2 = h_{m+r}h_{m-r}h_n^2 - h_{n+r}h_{n-r}h_m^2$ for every natural number $m \geq n \geq r$. The second definition says that a sequence of integers $\{\beta_n\}_{n\in\mathbb{N}}$ is an elliptic divisibility sequence if it is the sequence of the square roots (chosen with an appropriate sign) of the denominators of the abscissas $x(nP)$ of the iterates of a point P on a rational elliptic curve. It is well-known that the two definitions are not equivalent. Hence, given a sequence of the denominators $\{\beta_n\}_{n\in\mathbb{N}}$, in general does not hold $\beta_{m+n}\beta_{m-n}\beta_r^2 = \beta_{m+r}\beta_{m-r}\beta_n^2 - \beta_{n+r}\beta_{n-r}\beta_m^2$ for $m \geq n \geq r$.

During the talk, we will introduce the problem and we will prove that the recurrence relation above holds for $\{\beta_n\}_{n\in\mathbb{N}}$ under some conditions on the indexes m, n, and r.