Introduction

We begin with an activity to introduce the idea of dividing into districts.

Quick survey :

Ask the students the following question:

"What is your favorite food: pizza or hamburger?"

- Each student chooses:
  - P for pizza
  - H for hamburger

The teacher records the preferences of 25 students on the board, forming a **5×5 square grid** to represent 25 positions.

- The teacher then observes the grid and announces the **overall winner** by counting the total votes for each food and asking students to calculate **the percentage** of votes obtained by the winner.

Applying districts :

Now introduce the concept of electoral districts:

Divide the grid into districts of **5 squares:**

- By columns
- By rows

In each district, determine the **majority preference**.

Then count the number of **districts won** by each food.

Finally, compare the **direct vote result** with the district-based result to see if the winner remains the same or changes.

Modifying districts to influence results :

Finally, rearrange the districts with a goal:

- Make the less popular food win by creating strategic districts.
- This introduces **gerrymandering**: showing that by changing how groups are formed, the result of an election can be influenced—or even reversed.

Additional video

To explore further, students are asked to watch the following short video:

https://www.youtube.com/watch?v=bGLRJ12uqmk

Gerrymandering Level 1

Pair work and class discussion

Students discover that dividing votes into districts can influence election outcomes. Even if a party doesn't have the overall majority, it can win more districts and therefore the election.

Give students about 10 minutes to experiment in pairs with the grid —first dividing it vertically, then horizontally—to determine the winner in each scenario. Then spend about 5 minutes reviewing the results as a class.

Assume that nine people live in a territory represented by a grid. Here are the results of the last election between parties **A** and **B**.

A	B	A
B	A	B
A	A	B

How many votes did Party A get?	5 votes
How many votes did Party B get?	4 votes
Who won the election?	Party A

	Number of votes	Percentage
Party A	5	$\frac{5}{9} = 0{,}\overline{5} \sim 0{,}56 = 56\,\%$
Party B	4	$\frac{4}{9} = 0{,}\overline{4} \sim 0{,}44 = 44\,\%$

We decide to divide the 9 people into 3 groups of 3.

First, we divide the grid into three groups called districts, in vertical lines (each column is a district).

	D1	D2	D3
	A	B	A
	B	A	B
	A	A	B
Winner	A	A	B

If the election is won by the number of districts won, and not by the total number of votes, who won this election?

How many districts were won by Party A?	2 districts
How many districts were won by Party B?	1 district
Who won the election?	Party A

Now let's try to form the districts horizontally.

				Winner
D1	A	B	A	A
D2	B	A	B	B
D3	A	A	B	A

How many districts were won by Party A?	2 districts

How many districts were won by Party B?	1 district
Who won the election?	Party A

Is it possible to divide the territory that gives advantage to a particular party?

Students can propose several redistricting strategies, which are analyzed and discussed in groups. Once a suitable solution is found, the definition of gerrymandering is considered. This allows students to formalize what they have just discovered through experience. We recommend allowing approximately 15 minutes for this part.

Now choose for yourself how to divide the grid into three districts, respecting the following rules:

- Each district must contain exactly three squares.
- The cells of the same district must be connected by a side (horizontally or vertically, and not diagonally).

Try to create districts that favor Party B. Explain how different ways of creating districts can change the outcome of this election.

A	B	A
B	A	B
A	A	B

Party B can win the election by changing the way districts are drawn: it loses by a wide margin in one district, but barely wins the other two. This allows it to win the election, even though it has fewer total votes than Party A.

« Gerrymandering » = Electoral manipulation

Gerrymandering is when the boundaries of electoral districts are drawn in a certain way to help a party win. Even if this party does not have the majority of votes, it can win more districts by grouping the votes to its advantage.

Gerrymandering Level 2

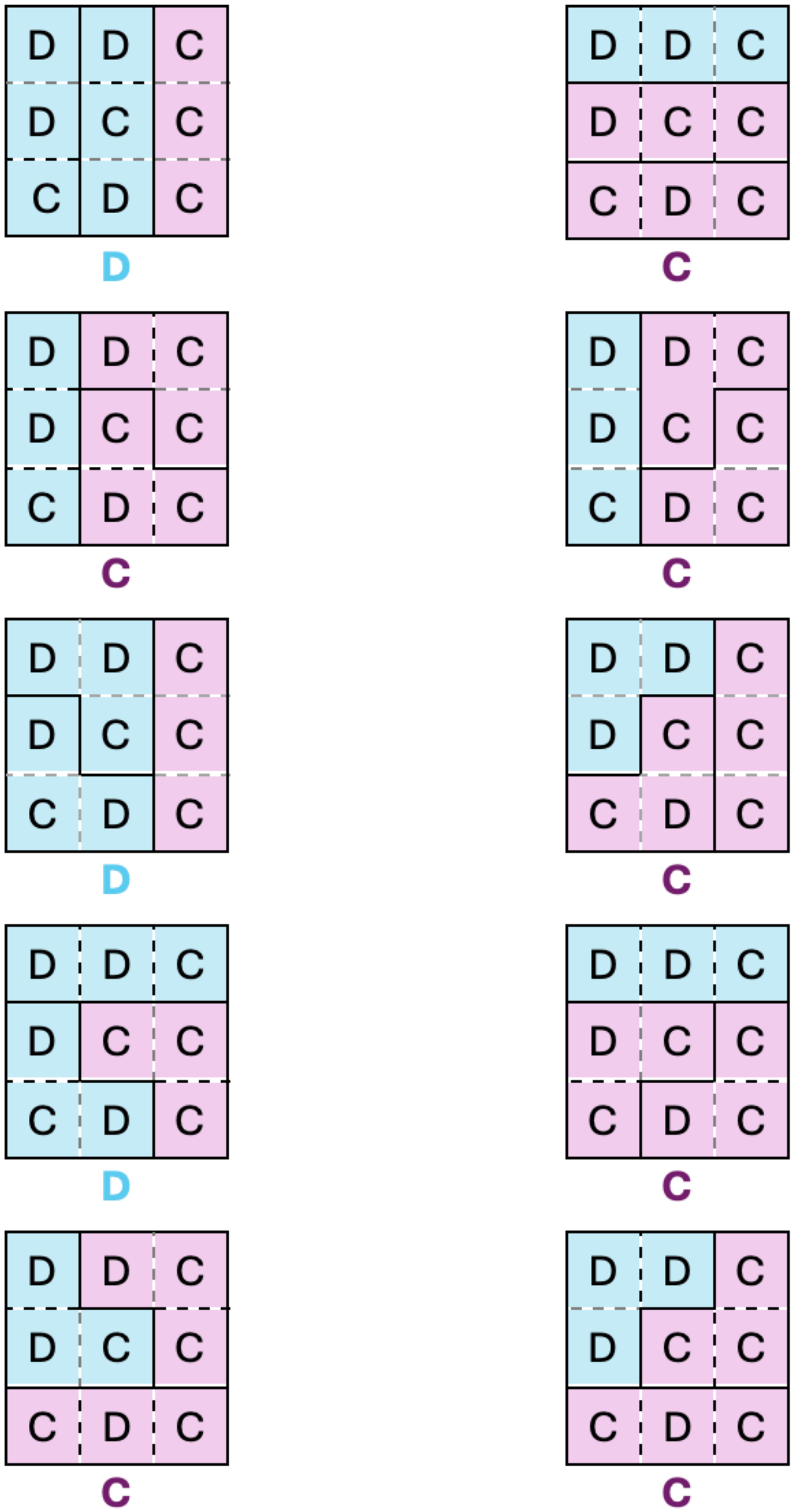
Individual work, then rapid correction in plenum

In this activity, it is suggested that students work individually for about 10 minutes to explore different ways of dividing the grid into districts of 3 voters. The goal is for them to realize that **three of these 10** configurations allow **party D**, despite being a minority in terms of votes, to win the election. A quick collective correction of around 5 minutes will generally be enough to visualize the 10 possibilities: you can ask a volunteer student to come and present on the board the cut-outs that they have found themselves.

Here is a new case where Party **C** got more votes than Party **D**. Your mission:

- Find as many ways as possible to create 3-vote districts (following the rules previously stated).
- For each proposition, clearly indicate the winner of the election (the party that wins the most districts).

D	D	C
D	C	C
C	D	C



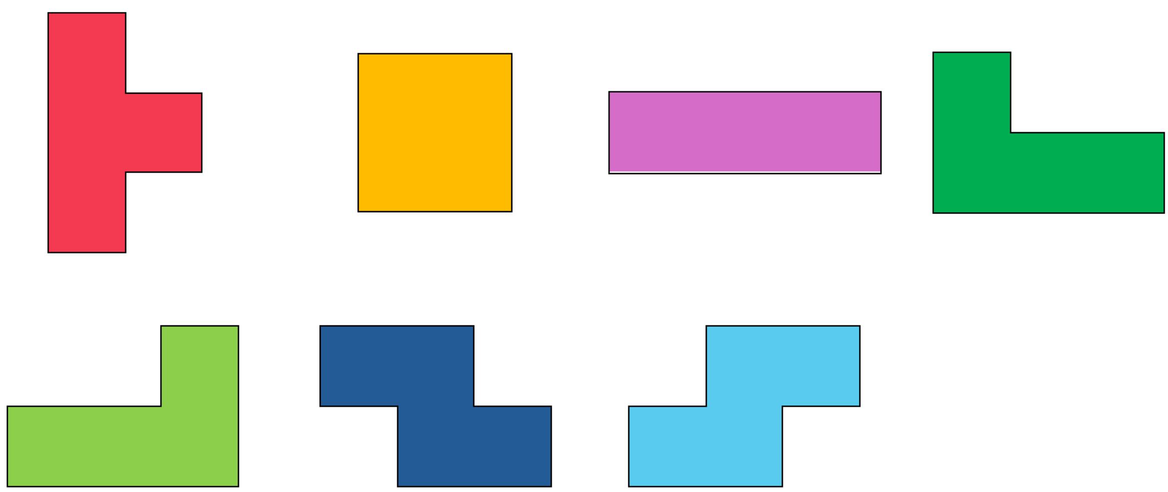
There are therefore **10** ways to divide the grid into districts. Among them, **3** out of **10** allow party D (even though it has a minority in terms of votes) to win the election.

Tetris-style district exploration

Individual work

For this activity, we suggest letting the students work individually for 10 to 15 minutes. Their mission is to create as many grids as possible using Tetris-type pieces (also called Tetrominos), each representing a district of 4 squares in a grid. The goal is to let their creativity run wild. At the end of the activity, students are asked to provide an estimate of the total number of possible divisions, as in the previous exercise with the 3x3 grid, where there were 10. The correct answer here is 117, which students probably won't find on their own. This is a good time to project the next page and ask them to compare their results with this list (the checkboxes allow you to see how many configurations the class has found). This activity fosters the awareness that by increasing the size of the grid the number of possible configurations increases very significantly, which makes electoral division more and more complex.

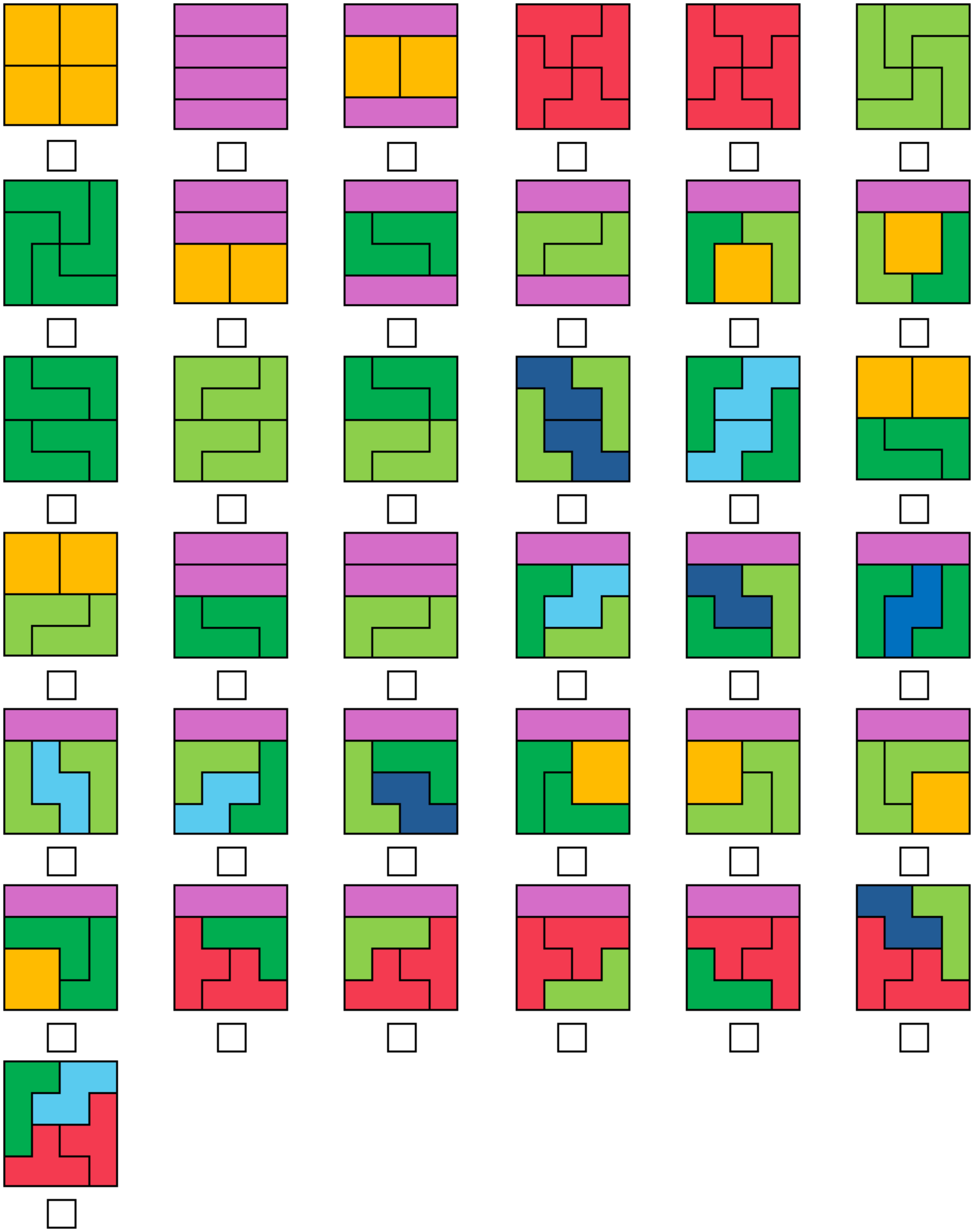
Now we work with a 4×4 grid. We decide to create districts containing 4 contiguous cells. Here are the possible shapes of districts up to rotation:



Can you determine many possible ways of decomposing the 4×4 grid with such districts?

Here are all the possibilities (not including rotations) for distributing districts on a 4x4 grid using Tetromino districts.

Gerrymandering Level 3



Work in pairs, then correction in plenary session.

For this level, students are invited to work in pairs again for about 10 minutes. Their mission is to distribute the votes of each voter in the grid themselves, so that **Julia**, despite having fewer votes than Alex, wins the election thanks to strategic districting.

This activity is a great opportunity to observe how students approach the challenge: will they choose a quick and easy method, or will they opt for a more elaborate strategy? The goal is for them to understand how it is possible to make a minority candidate win by distributing the votes optimally across districts. They should also realize that Julia can win the election **even with fewer votes**, provided that the electoral districting is well thought out, in other words, that gerrymandering is implemented.

For the correction (approximately 5 minutes), we recommend starting with the students' concrete proposals in order to enhance their reasoning, and to guide them towards the best strategy to make Julia win despite her initial disadvantage.

The 25 students in a class must elect their class representative. Two candidates are standing: Julia and Alex.

After the vote, the regent counts the ballots and obtains the following result:

- ➔ Alex got 13 votes
- ➔ Julia got 13 votes

Can you distribute the votes for Alex (A) and Julia (J) in the 5×5 grid below (boxes), such that, if each row is a district of 5 students, Julia wins the election?

A	A	A	A	A

A	A	J	J	J
A	A	J	J	J
A	A	J	J	J
A	A	J	J	J

Could Julia have won the election with fewer than 12 votes? What is the minimum number of votes Julia needs to win, using some strategic gerrymandering?

- There are **25** voters in total, spread over **5** districts.
- Julia needs to win **3** out of **5** districts to win the election.
- The minimum number of votes needed to win a district is **3**.
- So, in total, Julia needs at least **9** votes to be able to win the election, provided the districting allows it.

Generalization for odd numbers

Work in plenum, then in pairs and then return to plenum

At the end of this activity, students will have a general formula valid for all odd numbers.

Note: In the case of an even number of voters, the possibility of tie votes must also be provided for**.** See the website

https://math.uni.lu/voting

(French version, chapter Gerrymandering – Advanced Level)

for arbitrary numbers of voters and district sizes.

Number of voters in a district, number of districts	n odd	7
Minimum number of districts needed to win the election	$\frac{n+1}{2}$	4
Minimum number of votes needed in a district to win it	$\frac{n+1}{2}$	4
Minimum number of votes needed to win the election	$\left(\frac{n+1}{2}\right)\left(\frac{n+1}{2}\right)$	16

What is the minimum percentage required to win an election?

Some examples of the minimum number of votes required:

n	minimum number of votes to win	percentage of votes
3	4	56%
5	9	44%
7	16	39%
9	25	36%
11	36	34%
13	49	29%
15	64	28%
101	2601	25%

This activity allows you to better understand the formulas and to see that the percentage stabilizes around 25% when the number of voters becomes very large.

When the total number of voters becomes very large, what becomes the minimum percentage of votes needed to win the election?

We have

$$\left(\frac{n+1}{2}\right)\left(\frac{n+1}{2}\right) = \frac{1}{4}(n^2 + 2n + 1)$$

so the ratio with the total number of n^2 voters is

$$\frac{1}{4}\left(1 + \frac{2}{n} + \frac{1}{n}\right)$$

For large n, the last two terms of the sum are

then approximately $\frac{1}{4}$ i.e. $25\ \%$.

These terms are becoming increasingly negligible; this is why the percentages become increasingly smaller, but they remain above 25%.