MATH DAY 2023

INSTRUCTIONS:

Exercises

JUNIOR: Exercises 1 to 12 included.
INTERMEDIATE: Exercises 5 to 16 included.
SENIOR: Exercises 11 to 16 included and the three Proof Exercises.
WARNING Exercises beyond the requested ones will not be counted!

Grading

The exercises are multiple choice and graded as follows: 3 points for the correct answer, 0 points for the wrong answer, 1 point for no answer. Only one answer per question is correct.
Only for Senior: The proof exercises are graded with 9 points each. Written solutions have to be provided.

Please fill in the information below. Please write your answers to the multiple choice exercises into the table on the following page.

FIRST NAME

SURNAME

CATEGORY

(Junior/Intermediate/Senior)

Exercise	Category	Answer (A,B,C,D,or E)
1	J	
2	J	
3	J	
4	J	
5	J, I	
6	J, I	
7	J, I	
8	J, I	
9	J, I	
10	J, I	
11	J, I, S	
12	J, I, S	
13	I, S	
14	I, S	
15	I, S	
16	I, S	

Exercises

1. The twins Erik and Oskar have to share their favorite game for one year (January 1st to December 31st). The year is not a leap year. They decide that Erik will have the game on days which have an even number in the month (the 2nd of each month, the 4th of each month and so on) and Oskar on days which have an odd number in the month. However, they realize that there are more odd days than even days. In fact, how many more?

   A: 2

   B: 5

   C: 7

   D: 12

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2. Two gentlemen, Mr Adams and Mr Beckam, arrive at a door at the same time, and have to agree upon who enters first. At regular intervals, they make an attempt to find an agreement. At every attempt, they can speak (saying "You go first") or stay silent. If they both speak, nobody enters. If nobody speaks, nobody enters. If only one speaks, the silent gentleman enters.
   If they speak at the same time, Mr Adams will stay silent for at least one round, and Mr Beckam for at least two rounds. If nobody has spoken for at least 3 consecutive rounds, then Mr Beckam will speak in the next round. Neither gentleman knows the rules governing the other gentleman, and they do not agree on a strategy. Must there eventually come a round where one gentleman enters the door?

   A: Yes

   B: No

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3. There are four statues of an elf which only differ in their size. The sizes are XL (Extra Large), L (Large), M (Medium), S (Small) respectively. They have different weights decreasing in this order, i.e. the largest statue is the heaviest, the second-largest the second-heaviest etc. Your friend weighed the statues in pairs, and the weight of the various pairs, in grams, are

   $$18, 24, 30, 30, 36, 42\,.$$

   What is the total weight in grams of the four statues?

A: 60

B: 120

C: 150

D: 180

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4. A kid is playing with a car that can be remotely maneuvered. The car can only turn left or right at 90° angles. Moreover, it turns exactly once after each meter it has gone, and the kid can only decide whether it will be a left or a right turn. The kid is playing on a 3 meter × 4 meter rectangular carpet. Starting from one carpet corner parallel to one of the carpet sides, how many further carpet corners can be reached by the car?

   A: 1

   B: 2

   C: 3

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5. There is a world similar to ours, but where people are either liars or truth-tellers. Liars always lie and truth-tellers always tell the truth. A child says: "Everyone in my family is a liar". Is this assertion:

   A: True

   B: False

   C: Forcibly neither true nor false

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6. You are preparing a yoga session where the participants will be seated on chairs arranged in a row. The chairs are next to one another with no space in between. The participants must however be able to stretch their arms sideways without touching one another, therefore between any two occupied seats, there must be two empty seats. After setting up the row of chairs by taking all the available space, you decide to remove two chairs from one side because doing this does not decrease the number of seats available to the participants. What is the remainder after division by 3 of the number of chairs in the row, after having removed the two chairs?

   A: 0

   B: 1

   C: 2

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7. You are allowed to draw N balls from an urn containing many balls of each of the following colors: blue, green, yellow, red. You win once you draw 1 blue ball, or 2 green balls, or 3 yellow balls, or 4 red balls. What is the least possible value of N for which you are guaranteed to win?

   A: 4

   B: 5

   C: 6

   D: 7

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8. In a shop, they sell triangular tiles, where each tile has the side lengths 8,12, and 18 centimeters. You like the shape, but you need a larger tile. The triangle you want is similar to the ones sold at the shop and the side lengths, in centimeters, are again integers. Moreover, two of its side lengths are 12, and 18 centimeters. What is the third side length, in centimeters?

   A: 24

   B: 27

   C: 30

   D: 36

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9. Alice takes 4 hours to paint a fence while Bob takes 12 hours for the same task. How many hours will it take if they work together on the task?

   A: 1

   B: 2

   C: 3

   D: 4

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10. You are organizing a movie day at your school. You know that 50% of the pupils have watched "Aliens and Alligators" (movie A) and 40% of the pupils have watched "Basketball and Biscuits" (movie B). You know that 20% of the pupils who watched movie A also watched movie B. What percentage of the pupils who watched movie B also watched movie A?

A: 5%

B: 10%

C: 25%

D: 50%

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11. In your favorite restaurant, you can buy tofu nuggets to go. They are sold in packs of 3, 5, or 7. It is thus impossible to order, for instance, precisely 2 tofu nuggets. How many integers $n \geq 1$ exist for which you cannot buy precisely n nuggets?

    A: 1

    B: 2

    C: 3

    D: 4

    E: 5

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12. Your class is preparing for an excursion and your teachers are filling lunch bags with sandwiches, one lunch bag for each pupil. The lunch bags and the sandwiches are all alike. Each sandwich can be cut into 2 or 3 equal parts. You know that with 14 sandwiches, one can fill 9 lunch bags but not 10 lunch bags. How many sandwiches were used to prepare lunch bags for the 24 pupils?

    A: 24

    B: 30

    C: 32

    D: 36

    E: 48

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13. In a bag there are balls, each of which is coloured red, yellow or green. We pick two balls in a row, without putting the first one back. The probability of picking two red balls is $\frac{1}{7}$ and the one of picking two yellow balls is $\frac{1}{5}$. What is the least possible number of balls in the bag to make this true?

A: 15

B: 35

C: 70

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14. The new Luxembourgish car plates consist of two letters followed by a 4-digit number. As the letter O and the digit 0 look too similar on plates, one of the two symbols needs to be forbidden. Which symbol should we forbid to achieve the largest number of possible car plates? (The considered alphabet has 26 letters.)

    A: The letter O.

    B: The digit 0.

    C: The choice does not matter.

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15. How many integers $n \geq 1$ exist for which $16n^2 + 25$ is the square of a natural number?

    A: 0

    B: 1

    C: 2

    D: 5

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16. There are five small statues of an elf, and you know their combined weight W. The statues differ in their sizes, which are XL (Extra Large), L (Large), M (Medium), S (Small), XS (Extra Small) and they have different weights decreasing in this order. Your friend weighed the statues in pairs, and wrote the list of values

    $$W_1, W_2, \ldots, W_{10}$$

    from the largest to the smallest. Knowing only $W_2 + W_{10}$, which is the statue of which you can find out the weight?

    A: XL

    B: L

    C: M

    D: S

    E: XS

Proof Exercises, only for SENIOR

Problem 1. *Let $ABCD$ be a rectangle with $AB > BC$ and such that $BC = 1$. Let E be the point different from B such that $CE = BC$ and $\angle AEC = 90°$. Assume that $\angle DCE = 30°$. Find the length AB.*

Problem 2. *You are trying to crack a code of 8 digits. You know that the code represents the birthday of someone who was born between the year 1 (included) and today, in the format ddmmyyyy (for example, today's date would be 25022023). You also have the following information:*

- *The day is a prime number;*
- *the prime decomposition of the year is given by $p \cdot (p+10) \cdot (2p+3)$, where p is a prime number;*
- *the person was born on a Wednesday.*

What is the code?

Problem 3. *The following is an integer; find its value:*

$$2\sqrt{\frac{2\sqrt{5}-4}{\sqrt{5}+1}} + \sqrt[3]{17\sqrt{5}-38}.$$