MathDay 2022 Junior

Questions (16 multiple choice questions)

1. One of Alice, Bob, and Charles is a liar, the other two tell the truth.
   Alice said: I am not a liar.
   Bob said: Alice is not a liar.
   Charles said: Alice is a liar.
   Who is the liar?

   Correct answer: Charles
   Answers: Alice,Bob,Charles
   Solution: Since Bob and Charles say contradictory things, one of them is a liar. So Alice cannot be the liar. Hence Charles is the liar.

2. Bob's grandparents like roses. On some days they want 3 white roses, 2 red roses, and 1 yellow rose. On the other days, they want 1 white rose, 2 red roses, and 3 yellow roses. Bob is at the flower shop and forgot his grandparents' preference for today. Bob needs to buy enough roses to be sure that the right ones are among them (the number of roses he buys may depend on the color, as one can buy single roses of any color). What is the least number of roses he needs to buy?

   Correct answer: 8
   Answers: 6, 8, 10, 12
   Solution: Bob needs 3 white roses, 2 red roses, and 3 yellow roses.

3. In a board game there are many direction cards, which are either North, East, West, or South. A player that has three direction cards of the same type wins. How many cards suffice for a player to win with absolute certainty? Select the smallest possible number.

   Correct answer: 9
   Answers: 3, 6, 9, 12
   Solution: The most unlucky case is having two cards of each type, namely 8 cards. With 9 cards one is sure to have three cards of the same type.

4. Forty children received an invitation to a birthday party, but not all of them came. At the party, the children played a game for teams of 8 players and no child was left without a team. They also played a game for teams of 5 players; two children were

left without a team and they became referees for this game. How many children were at the party?

Correct answer: 32
Answers: 12,20,24,32
Solution: We look for a number from 0 to 40 which is a multiple of 8 and which leaves remainder 2 after division by 5. The only such number is 32.

5. You have two identical apples, one banana, and one orange. You have to give them to four children (Alice, Bob, Charles, and David), so that each child receives exactly one piece of fruit. In how many different ways can you distribute the pieces of fruit to the children?

   Correct answer: 12
   Answers: 4, 6, 8, 12
   Solution: You only have to choose whom to give the banana to (4 choices) and whom the orange (3 choices left). There are 12 possibilities.

6. The frame for a painting should be in the form of a rectangle with a rectangular hole inside. To build such a frame you are given 4 pieces of wood. They all are 10 cm wide, but two of them are 40 cm long while two of them are 60 cm long. You can build different frames out of them by gluing the pieces of wood together. What is the largest hole that you can obtain inside the frame? Consider your result in square centimeters.

   Correct answer: 1600
   Answers: 1000, 1200, 1500, 1600
   Solution: You can build a 60cm$\times$60cm frame with a 40cm$\times$ 40cm hole (1600 square centimeters). Or you can make a 50cm$\times$70cm frame with a 30cm$\times$ 50cm hole (1500 square centimeters). Or you can make a 40cm$\times$80cm frame with a 20cm$\times$ 60cm hole (1200 square centimeters).

7. You have to solve 200 mathematical problems, and fortunately the genie of the lamp will help you by fulfilling some wishes. Wish 1 will let you solve 60% of the unsolved problems, Wish 2 will let you solve 40% of the unsolved problems, Wish 3 will let you solve 50 unsolved problems (or all remaining problems, if you have less than 50 unsolved problems left). You can ask two distinct wishes in the order you prefer. Which wishes should you ask to solve most of the problems? For example, the answer (2,3) means that the first wish you ask is Wish 2 and the second wish you ask is Wish 3.

   Correct answer: (1,3)
   Answers: (1,2), (2,1), (1,3), (3,1)
   Solution: (1,2) and (2,1) will both leave 48 problems unsolved. (1,3) will leave 30 problems unsolved. (3,1) will leave 60 problems unsolved. (2,3) and (3,2) solve less problems than (1,3) and (3,1) respectively.

8. You are driving a remote control toy car on a circuit in the shape of a regular polygon with 10 sides. At the end of each straight segment you turn left at an angle which is strictly between 0 and 180 degrees, and how much you turn is the measure of this angle. How much do you need to turn in total while doing one round on the full circuit, starting from the middle of a side? The answer is in degrees.

   Correct answer: 360
   Answers: 180,360,900,1800
   Solution: The sum of the exterior angles of any regular polygon is 360 degrees.

9. In a foreign land there is a currency called AUR. There are golden coins with value 1AUR, 3AUR and 9AUR. What is the smallest number of coins you need to be able to pay any bill in the range from 1AUR to 44AUR? You can choose the coins freely, but you have to choose them before knowing the amount of the bill.

   Correct answer: 8
   Answers: 7,8,9,10
   Solution: You can take 4 9AUR coins, 2 3AUR coins, 2 1AUR coin. This is optimal because with only 3 or fewer coins of 9AUR, you would need at least 10 coins to pay 44 AUR (3 x 9AUR + 5 x 3AUR + 2 x 1AUR), and with 4 coins of 9 AUR you need at least 8 coins to pay 44 AUR. The given coins allow you to pay any bill in the range; using the 9AUR coins one is left to pay an amount from 1AUR to 8AUR, which can be paid with the given 3AUR or 1AUR coin.

10. Alice and Zoe, when they run alone, always run at their usual constant speed. Alice runs 1 kilometer in 3:20 (3 minutes and 20 seconds), while Zoe runs 1 kilometer in 4:10. They planned to run together on a 9 km long straight path along the river. But they just texted each other and found that, due to a misunderstanding, they are at opposite ends of the path. Now they start running towards each other. After how much time will they meet?

    Correct answer: 16:40
    Answers: 12:30, 13:20, 16:40, 20:00
    Solution: Alice runs 1km in 200 seconds, Zoe in 250 seconds, so Alice's speed is 5/4 of Zoe's speed. Hence they will meet after Alice made 5/9 of the total distance and Zoe 4/9 of the total distance. Thus Alice has run 5km, which will take 16:40.

11. You have 8 sticks to make a closed polygon, and the sides of the polygon must be either horizontal or vertical. If each stick has length 1, what is the largest area that you can obtain by using all sticks? As unit measure for the area consider the square of the unit length.

    Correct answer: 4
    Answers: 3,4,5,6
    Solution: Using all sticks, you can build a 2×2 square with area 4, a 3×1 rectangle with area 3, and an L-shaped figure with area 3.

12. There are two fair dice with four faces. The first die is red and the second die is blue, and you throw them both at once. On the four faces of each die there are the numbers from 1 to 4. What is the most likely sum of the results of the two dice?

    Correct answer: 5
    Answers: 4,5,6,7
    Solution: Consider the set of results of the first die and the second die, consisting of the 16 pairs made with the numbers from 1 to 4. The numbers 2 and 8 can be obtained each just with one pair. The numbers 3 and 7 can be obtained each with two pairs. The number 4=3+1=2+2 and 6=4+2=3+3 can be obtained each with three pairs. The number 5=4+1=3+2 can be obtained with four pairs.

13. You are in a video call with some friends who are native speakers of the Combish language. You only remember four different words in this language: Xix, Yiy, Ziz, Wiw. Exactly one of them is extremely funny. You know that if you send some of these words to any of your friends, then that friend will start laughing immediately if and only if the funny word is among the words you chose.

    You can write exactly one message with some Combish words to each of your friends in the call. You can choose how many words and which words to write. You can send different individual messages, but all messages are sent at the same time. You are then able to check in the video call who is laughing. What is the minimum number of friends that you need in the call so that you are able to determine without doubt the funny word by using the above method?

    Correct answer: 2
    Answers: 1,2,3,4
    Solution: One friend is not enough because from testing only one message you cannot determine the funny word in all possible cases. Two friends are enough: you send the first word only to the first friend, the second word only to the second, the third word to both, and the fourth word to none.

14. In a group of koalas, the two lightest koalas together weigh 25% of the total weight of the group, while the three heaviest koalas together weigh 60% of the total weight of the group. How many koalas are in the group?

    Correct answer: 6
    Answers: 5, 6, 7, 8
    Solution: More than 5 because the five mentioned koalas do not make up the 100% of the weight. Each intermediate koala weights each at least 12,5% of the total weight and at most 20% of the total weight. Since only 15% of the total weight is missing, there is space for at most one intermediate koala. So there are 6 koalas.

15. Amy and Ben play the Candy Game. At the beginning of the game, there are 10 candies. Amy and Ben take turns making moves. A move consists of removing

either 2 or 3 candies. The first player that cannot make a move (because there are less than 2 candies left) loses. Amy makes the first move. If both Amy and Ben aim to win and play according to the best possible strategy, who wins the game?

Correct answer: Ben
Answers: Amy, Ben
Solution: Consider the number of remaining candies. When you see 0 or 1 candies on the table, you lose. When there are 2,3 or 4 candies, you win (for 4 candies take 3 candies, for 3 candies take 2 or 3 candies, for 2 candies take 2 candies). When there are 5 or 6 candies, you lose (taking either 2 or 3 candies puts the other player in a winning situation). When there are 7,8 or 9 candies you win (taking either 2 or 3 candies puts the other player in a losing situation). When there are 10 candies, you lose. (taking either 2 or 3 candies puts the other player in a winning situation). So the second player always wins, i.e. Ben.

16. What is the maximum number of bishops you can place on a 4×4 chessboard so that no two bishops are on the same diagonal line?

Correct answer: 6
Answers: 4,5,6,7
Solution: A1, A3, B1, C4, D1, D2 works, so the answer is at least 6. There are 7 diagonals going bottom left to top right, so the answer is at most 7. However the first and last diagonal are on a same diagonal line in the other direction, so you cannot place bishops on both of them. Thus the answer is at most 6.