MathDay 2022 Intermediate

Questions (12 questions without proof)

Each question is worth 3 points. The correct answer gives 3 points, the wrong answer gives 0 points, not answering gives 1 point.

1. A fairground booth boasts a game involving the wheel of fortune. The numbers 1 to 5 are written on the wheel, and when it is spun, every number occurs with equal probability.

   To play the game, you place a bet on any number between 2 and 10 (inclusive) of your choice. The wheel is then spun twice and if the sum of the two resulting numbers equals the number you bet on, you win a teddy-bear. Otherwise, you don't win anything. What number should you bet on to maximize your chance of winning the teddy-bear?

2. What is the maximum number of queens you can put on a 5×5 chessboard, so that no two queens are in the same diagonal, row or column?

3. Sixty children received an invitation to a birthday party, but not all of them came. At the party, the children played a game for teams of 12 players and no child was left without a team. They also played a game for teams of 5 players; one child was left without a team and became referee for this game. How many children were at the party?

4. You have two identical apples, two identical oranges, and one banana. You have to give them to five children, so that each child receives exactly one piece of fruit. In how many different ways can you distribute the pieces of fruit to the children?

5. In a foreign land there is a currency called AUR. There are golden coins with values 1AUR, 3AUR and 9AUR. What is the smallest number of coins you need to be able to pay any bill in the range from 1AUR to 107AUR? You can choose the coins freely, but you have to choose them before knowing the amount of the bill.

6. Alice and Zoe, when they run alone, always run at their usual constant speed. Alice runs 1 kilometer in 4:10 (4 minutes 10 seconds), while Zoe runs 1 kilometer in 5:00. They planned to run together on a 11 km long straight path along a river. But they just texted each other and found that, due to a misunderstanding, they are at opposite ends of the path. Now they start running towards each other. After how much time will they meet? Give your answers in minutes.

7. You arrive on an island that is inhabited by 7 dwarves. A dwarf can either be a truth-teller or a liar. The truth-tellers always speak the truth and the liars always lie. All dwarves queue in a straight line to greet you. They all look into your direction.
   The first dwarf in the line says: "All dwarves behind me are liars."
   All other dwarves say: "The dwarf right in front of me is a liar."
   How many dwarves are liars?

8. You are in a video call with some friends who are native speakers of the Combish language. You only remember four different words in this language: Xix, Yiy, Ziz, Wiw. Exactly one of them is extremely funny. You know that if you send some of these words to any of your friends, then that friend will start laughing immediately if and only if the funny word is among the words you chose.

   You can write exactly one message with some Combish words to each of your friends in the call. You can choose how many words and which words to write. You can send different individual messages, but all messages are sent at the same time. You are then able to check in the video call who is laughing. What is the minimum number of friends that you need in the call so that you are able to determine without doubt the funny word by using the above method?

9. Amy and Ben play the Candy Game. At the beginning of the game, there are 10 candies. Amy and Ben take turns making moves. A move consists of removing either 2 or 3 candies. The first player that cannot make a move (because there are less than 2 candies left) looses. Amy makes the first move. If both Amy and Ben aim to win and play according to the best possible strategy, who wins the game? Answer 1 for Amy and 2 for Ben.

10. A very modern museum of very modern art has two floors, one on top of the other. Each floor consists of four corridors connected in the form of a square; it is possible to go from each corridor to the two neighbouring ones on the same floor; at the end of each corridor there is a staircase connecting vertically the two floors; the only entrance is also the only exit and it is located at one corner on the ground floor. You want to walk along each corridor exactly once (the direction doesn't matter to you). You can walk along different tours, according to the order in which you visit the corridors. How many different tours are there, assuming that you use the stairs exactly twice?

11. You have a coin that when being thrown comes up heads more often than tails. You and a friend play the following game: You toss the coin twice. If the result is twice the same, you win. If the results of the tosses are different, your friend wins. Who is more likely to win? Answer 1 if you have a higher probability to win, answer 2 if your friend has a higher probability to win, answer 3 if you and your friend have the same probability to win.

12. You own a bag of letter-shaped pasta for children. There are 26 different letters. If you take 99 pieces of pasta from the bag, what is the largest integer number n such that you can be sure to have at least n pieces representing the same letter?