Mathday 2022 Junior

Questions (16 multiple choice questions)

Each question is worth 3 points. The correct answer gives 3 points, the wrong answer gives 0 points, not answering gives 1 point. Exactly one of the proposed answers is correct.

1. One of Alice, Bob, and Charles is a liar, the other two tell the truth.

Alice said: I am not a liar. Bob said: Alice is not a liar. Charles said: Alice is a liar.

Who is the liar?

Answers: Alice, Bob, Charles

2. Bob's grandparents like roses. On some days they want 3 white roses, 2 red roses, and 1 yellow rose. On the other days, they want 1 white rose, 2 red roses, and 3 yellow roses. Bob is at the flower shop and forgot his grandparents' preference for today. Bob needs to buy enough roses to be sure that the right ones are among them (the number of roses he buys may depend on the color, as one can buy single roses of any color). What is the least number of roses he needs to buy?

Answers: 6, 8, 10, 12

3. In a board game there are many direction cards, which are either North, East, West, or South. A player that has three direction cards of the same type wins. How many cards suffice for a player to win with absolute certainty? Select the smallest possible number.

Answers: 3, 6, 9, 12

4. Forty children received an invitation to a birthday party, but not all of them came. At the party, the children played a game for teams of 8 players and no child was left without a team. They also played a game for teams of 5 players; two children were

left without a team and they became referees for this game. How many children were at the party?

Answers: 12,20,24,32

5. You have two identical apples, one banana, and one orange. You have to give them to four children (Alice, Bob, Charles, and David), so that each child receives exactly one piece of fruit. In how many different ways can you distribute the pieces of fruit to the children?

Answers: 4, 6, 8, 12

6. The frame for a painting should be in the form of a rectangle with a rectangular hole inside. To build such a frame you are given 4 pieces of wood. They all are 10 cm wide, but two of them are 40 cm long while two of them are 60 cm long. You can build different frames out of them by gluing the pieces of wood together. What is the largest hole that you can obtain inside the frame? Consider your result in square centimeters.

Answers: 1000, 1200, 1500, 1600

7. You have to solve 200 mathematical problems, and fortunately the genie of the lamp will help you by fulfilling some wishes. Wish 1 will let you solve 60% of the unsolved problems, Wish 2 will let you solve 40% of the unsolved problems, Wish 3 will let you solve 50 unsolved problems (or all remaining problems, if you have less than 50 unsolved problems left). You can ask two distinct wishes in the order you prefer. Which wishes should you ask to solve most of the problems? For example, the answer (2,3) means that the first wish you ask is Wish 2 and the second wish you ask is Wish 3.

Answers: (1,2), (2,1), (1,3), (3,1)

8. You are driving a remote control toy car on a circuit in the shape of a regular polygon with 10 sides. At the end of each straight segment you turn left at an angle which is strictly between 0 and 180 degrees, and how much you turn is the measure of this angle. How much do you need to turn in total while doing one round on the full circuit, starting from the middle of a side? The answer is in degrees.

Answers: 180,360,900,1800

9. In a foreign land there is a currency called AUR. There are golden coins with value 1AUR, 3AUR and 9AUR. What is the smallest number of coins you need to be able

to pay any bill in the range from 1AUR to 44AUR? You can choose the coins freely, but you have to choose them before knowing the amount of the bill.

Answers: 7,8,9,10

10. Alice and Zoe, when they run alone, always run at their usual constant speed. Alice runs 1 kilometer in 3:20 (3 minutes and 20 seconds), while Zoe runs 1 kilometer in 4:10. They planned to run together on a 9 km long straight path along the river. But they just texted each other and found that, due to a misunderstanding, they are at opposite ends of the path. Now they start running towards each other. After how much time will they meet?

Answers: 12:30, 13:20, 16:40, 20:00

11. You have 8 sticks to make a closed polygon, and the sides of the polygon must be either horizontal or vertical. If each stick has length 1, what is the largest area that you can obtain by using all sticks? As unit measure for the area consider the square of the unit length.

Answers: 3,4,5,6

12. There are two fair dice with four faces. The first die is red and the second die is blue, and you throw them both at once. On the four faces of each die there are the numbers from 1 to 4. What is the most likely sum of the results of the two dice?

Answers: 4,5,6,7

13. You are in a video call with some friends who are native speakers of the Combish language. You only remember four different words in this language: Xix, Yiy, Ziz, Wiw. Exactly one of them is extremely funny. You know that if you send some of these words to any of your friends, then that friend will start laughing immediately if and only if the funny word is among the words you chose.

You can write exactly one message with some Combish words to each of your friends in the call. You can choose how many words and which words to write. You can send different individual messages, but all messages are sent at the same time. You are then able to check in the video call who is laughing. What is the minimum number of friends that you need in the call so that you are able to determine without doubt the funny word by using the above method?

Answers: 1,2,3,4

14. In a group of koalas, the two lightest koalas together weigh 25% of the total weight of the group, while the three heaviest koalas together weigh 60% of the total weight of the group. How many koalas are in the group?

Answers: 5, 6, 7, 8

15. Amy and Ben play the Candy Game. At the beginning of the game, there are 10 candies. Amy and Ben take turns making moves. A move consists of removing either 2 or 3 candies. The first player that cannot make a move (because there are less than 2 candies left) loses. Amy makes the first move. If both Amy and Ben aim to win and play according to the best possible strategy, who wins the game?

Answers: Amy, Ben

16. What is the maximum number of bishops you can place on a 4×4 chessboard so that no two bishops are on the same diagonal line?

Answers: 4,5,6,7