

Martin Schlichenmaier
Université du Luxembourg

Basic Algebraic Structures

Linear Codes

I. Parity Code $[4, 3]$

Generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

8 Codewords

$$\begin{array}{cccc} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \end{array}$$

Code distance = 2 (hence no error-correcting)

Control matrix H ($G \cdot {}^tH = 0$)

$$H = (1 \ 1 \ 1 \ 1)$$

Possible syndromes: 0 and 1.

II. Hamming Code $[7, 4, 3]$

Generator matrix

$$G = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

16 **Codewords**

0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1
1	0	0	1	1	0	0	0	0	1	1	0	0	1	1	1
1	0	0	0	0	1	1	1	0	1	1	1	1	0	0	0
0	1	0	1	0	1	0	0	1	0	1	0	1	0	1	1
0	1	0	0	1	0	1	1	1	0	1	1	0	1	0	0
0	0	1	1	0	0	1	0	1	1	0	0	1	1	0	0
0	0	1	0	1	1	0	0	1	1	0	1	0	0	1	1

Code distance = 3 (hence 1 error-correcting)

Control matrix H ($G \cdot {}^tH = 0$)

$$H = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

II. Hamming Code $[7, 4, 3]$

List of conductors and syndromes:

conductor e	syndrome $s(e)$
0000000	000
0000001	001
0000010	010
0000100	100
0001000	111
0010000	110
0100000	101
1000000	011

The web-page of the course:

<http://math.uni.lu/schlichenmaier/cours/mics>