SL2R Days

June 6–7, 2024, University of Luxembourg

Schedule

Thursday June 6, 2024 **(Room MSA 3.370, Campus Belval)**

13:00	Registration
13:45–14:45	**Emmanuel Pedon** *Continued fractions and Hankel determinants for q-metallic numbers*
14:45–15:45	**Wolfgang Bertram** *On Group and Loop Spheres*
15:45–16:15	Coffee break
16:15–17:15	**Wai Yeung Lam** *Discrete hyperbolic Laplacian*
17:15–18:15	**Benjamin Delarue** *Axiom A flows for projective Anosov subgroups*
19:30	Conference dinner at restaurant Nonbe *31 Porte de France, L-4360 Belval*

Friday June 7, 2024 **(Room MSA 3.370, Campus Belval)**

09:00–10:00	**Anke Pohl** *Divisor of the Selberg zeta function with unitary representations*
10:00–10:30	Coffee break
10:30–11:30	**Andrea Santi** *On 3-nondegenerate CR manifolds in dimension 7*
11:30–12:30	**Christian Arends** *Explicit Hilbert spaces for the unitary dual of rank one orthogonal groups*
12:45	Walking lunch near the conference room

- **Christian Arends** (Aarhus University):

Explicit Hilbert spaces for the unitary dual of rank one orthogonal groups

We realize all irreducible unitary representations of the group $\mathrm{SO}_0(n+1,1)$ on explicit Hilbert spaces of vector-valued L^2-functions on $\mathbb{R}^n \setminus \{0\}$. The key ingredient in our construction is an explicit expression for the standard Knapp--Stein intertwining operators between arbitrary principal series representations in the so-called F-picture which is obtained from the non-compact picture on a maximal unipotent subgroup $N \simeq \mathbb{R}^n$ by applying the Euclidean Fourier transform. As an application, we describe the space of Whittaker vectors on all irreducible Casselman--Wallach representations. Moreover, the new realizations of the irreducible unitary representations immediately reveal their decomposition into irreducible representations of a parabolic subgroup, thus providing a simple proof of a recent result of Liu--Oshima--Yu.

- **Wolfgang Bertram** (Université de Lorraine - Nancy):

On Group and Loop Spheres

The fact that the 1-sphere (circle S^1) is a (commutative) group is of basic importance for Euclidean geometry, for complex analysis, and for mathematics in general. Let us call "sphere" the level set $q(x) = const$ of a general quadratic form $q : V \to K$, and call it "group sphere" if it carries some "natural" group structure. The Euclidean spheres S^n are group spheres if, and only if, n equals $0, 1$ or 3. Our friend $SL(2,R)$ is a group sphere, too, whose quadratic form is the determinant map, $q(X) = det(X)$. What can one say in the general case: which spheres are group spheres? First of all, we have to give a precise definition of what "natural" group structure means in this context (one should notice that the choice of the unit element is arbitrary and "ungeometric"). This leads us to the definition of "spherical quadratic space". Then the first result is completely general: Every 2-dimensional quadratic space is spherical (here, K may be any base field or even ring). In higher dimension, the situation becomes more complicated: the rank of the quadratic space must necessarily be 4, but it is not true that every quadratic space of rank 4 is spherical. Finally, in dimension 8 there exist spheres that are "almost" group spheres, but they lack associativity: they are "loop spheres". I will present some new results (work in progress), giving a more geometric and group-theoretic view on issues that are closely related to the classical theory of composition algebras , and to the Albert-Cayley-Dickson construction.

- **Benjamin Delarue** (Universität Paderborn):

Axiom A flows for projective Anosov subgroups

Projective Anosov subgroups are higher rank generalizations of fundamental groups of convex cocompact hyperbolic manifolds. Key properties of these groups are closely related to the dynamical system given by the geodesic flow and in particular its restriction to its non-wandering set. In the projective Anosov case the latter dynamical system can be replaced by Sambarino's refraction flow (or a general Gromov geodesic flow). In my talk I will present recent joint work with Daniel Monclair and Andrew Sanders in which we show that every projective Anosov subgroup comes with an analytic contact Axiom A flow whose restriction to its non-wandering set becomes Sambarino's refraction flow. We deduce a general exponential mixing result and the existence of a discrete spectrum of Ruelle-Pollicott resonances with associated (co-)resonant states. If time allows, I will outline applications of our results to pseudo-Riemannian manifolds and Benoist subgroups.

- **Wai Yeung Lam** (Université du Luxembourg):

Discrete hyperbolic Laplacian

The Laplace operator on a Riemannian manifold is a fundamental tool to study the geometry of the manifold. Inspired by electric networks, Laplacians on graphs are defined with edge weights playing the role of conductance. When the edge weights are constant, the graph Laplacian becomes the combinatorial Laplacian and is known to reveal rich combinatorial information of the graph. Given a graph embedded on a surface, it is natural to consider a geometric Laplacian, where edge weights are adapted to the geometry. For the 1-skeleton graph of a geodesic triangulation on a Euclidean surface, there is a “cotangent formula” relating the edge weights to the Euclidean metric. It is known to connect with various problems, e.g. deformations of circle patterns, Delaunay decomposition, discrete harmonic maps and the Y-Delta transform in graphs. In the talk, we introduce the analogue for hyperbolic surfaces.

- **Emmanuel Pedon** (Université de Reims):

Continued fractions and Hankel determinants for q-metallic numbers

The 'metallic numbers', or 'metallic means', are the real numbers whose regular continued fraction expansion has the form $n+1/(n+1/(n+1/(n+\ldots$ for some positive integer n; the golden ratio (corresponding to $n=1$) is certainly the most famous example. My talk is devoted to their 'q-deformation' in the sense of S. Morier-Genoud and V. Ovsienko. I will present several continued fractions which characterize q-metallic numbers and show, when $n=1$ or $n=2$, that their Hankel determinants have amazing properties: they consist of $-1, 0$ and 1 only; they are periodic; and they form Somos or Gale-Robinson recurrences. I will explain why these properties make q-metallic numbers resembling Catalan and Motzkin numbers.
The talk is based on joint work with Valentin Ovsienko.

- **Anke Pohl** (Universität Bremen):

Divisor of the Selberg zeta function with unitary representations

The classical Selberg zeta function is a mediator between spectral entities and dynamical entities of hyperbolic surfaces, as it is defined by means of the geodesic length spectrum and encodes in its zeros the spectral parameters of the Laplacian of the considered hyperbolic surface. We will consider the Selberg zeta function of infinite-area, geometrically finite hyperbolic orbisurfaces with twists by finite-dimensional unitary representations and hence for vector-valued situations. We will present a factorization formula in terms of the Weierstrass product of the Laplace resonances, Barnes G-functions, gamma functions and the singularity degrees of the representation. Similar to the classical, untwisted case, this provides a spectral and geometric interpretation of the zeros and poles of the Selberg zeta function, but this time by spectral and geometric entities of the orbisurface and by the representation. We will see that this factorization formula generalizes the factorization result by Borthwick, Judge and Perry to hyperbolic orbisurfaces with orbifold singularities as well as to unitary twists. We will further see that the presence of orbifold singularities yields a separate, previously unobserved contribution to the factorization formula, even in the untwisted case. This is joint work with Moritz Doll.

- **Andrea Santi** (Università degli Studi di Roma Tor Vergata):

On 3-nondegenerate CR manifolds in dimension 7

The classification of locally homogeneous 2-nondegenerate CR-manifolds in dimension 5 was achieved in 2008 by the celebrated work of Fels--Kaup. The locally homogeneous model for such CR-manifolds is given by the tube over the future light cone $\mathcal{M}^5 \subseteq \mathbb{C}^3 = \mathbb{R}^3 + i\mathbb{R}^3$, which has been extensively investigated from the point of view of both complex analysis and physics.
I will report on joint works with B. Kruglikov on CR-hypersurfaces in $\mathbb{C}^4$ with a degenerate Levi form. I will discuss the symmetry dimension bound for all the 3-nondegenerate 7-dimensional CR real-analytic structures in the context of Beloshapka's conjecture, and present the classification of those that are homogeneous. The bound is achieved on the model $\mathcal{M}^7 \subseteq \mathbb{C}^4$, which has automorphism group $\mathrm{Aut}(\mathcal{M}^7) \cong \mathrm{GL}_2(\mathbb{R}) \ltimes S^3\mathbb{R}^2$ and it is locally the only homogeneous 3-nondegenerate 7-dimensional CR-manifold.