Analyse 1

HOMEWORK 1 : DUE THURSDAY OCTOBER 6 IN CLASS

Exercice I

If r is rational and x is irrational, show that both rx and $r+x$ are irrational.

Exercice II

If A, B are two nonempty subsets of $\mathbb{R}$ such that A is contained in B and B is bounded from above. Show that $\sup A \leq \sup B$.

Exercice III

Use mathematical induction to prove that for any integer $n \geq 1$,

$$1+2+\cdots+n=\frac{n(n+1)}{2}$$

and

$$1+2^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$$

Exercice IV

1. Prove that if a, b are positive real numbers, then $\sqrt{a+b} \leq \sqrt{a}+\sqrt{b}$.
2. Fix an integer $n \geq 2$, prove that for any $a_1, a_2, \cdots, a_n \geq 0$, $\sqrt{a_1+\cdots+a_n} \leq \sqrt{a_1}+\cdots+\sqrt{a_n}$.
3. Fix an integer $n \geq 2$, prove that for any $a_1, a_2, \cdots, a_n > 0$,

$$(a_1+\cdots+a_n)\left(\frac{1}{a_1}+\cdots+\frac{1}{a_n}\right) \geq n^2 .$$

Exercice V

Assume that A, B are two **nonempty** subsets of $\mathbb{R}$, we define

$$A+B:=\{a+b : a \in A \text{ AND } b \in B\} .$$

Show that if A, B are bounded from above, then

1. $A+B$ is bounded from above.
2. $\sup(A+B)=\sup A+\sup B$.