Analyse 1

HOMEWORK 1: DUE THURSDAY OCTOBER 20 IN CLASS

Exercice I

Find the limits of the following sequences. Notice that for this question, you only need write down the main steps without too much details.

1.
$$\lim_{n\to+\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\cdots+\frac{n-1}{n^2}\right)$$

2.
$$\lim_{n\to+\infty}\frac{(n+1)(n+2)(n+3)}{n^3}$$

3.
$$\lim_{n\to+\infty}\left(\frac{1+3+5+\cdots+(2n-1)}{n+1}-\frac{2n+1}{2}\right)$$

4.
$$\lim_{n\to+\infty}\frac{4^{n+1}+3^{n+1}}{5^n+3^n}$$

5.
$$\lim_{n\to+\infty}\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}\right)$$

6.
$$\lim_{n\to+\infty}\frac{1+4+9+\cdots+n^2}{n^3}$$

7.
$$\lim_{n\to\infty}\frac{\cos(n!)}{n^2}$$ [1]

8.
$$\lim_{n\to\infty}\frac{\sin(n^2)+n}{n+\arctan(n)}$$

Hint: *for question 1, 3, 6, you may want to use the results from the first homework.*

Attention: *for question 1, 3, 6, you are only required to solve ONE of THEM. You are very encouraged to finish all of them.*

Exercice II

Notice that for this question, you need justify your answer with enough details.

(i) If the sequence $(s_n, n \geq 1)$ converges to $s \in \mathbb{R}$, find the limit of the sequence $\big(|s_n|, n \geq 1\big)$.

(ii) If $s_1 = \sqrt{2}$ and $s_{n+1} = \sqrt{2+\sqrt{s_n}}$ for $n \geq 1$, this defines a sequence of positive numbers $(s_n, n \geq 1)$. Show that this sequence is convergent and find its limit.

(iii) If $u_n = \sqrt{n^2+2n} - n$, find the limit of $(u_n, n \geq 1)$, when $n \to +\infty$.

Exercice III

Let x_n be a sequence of real numbers. Prove that x_n converges to $x \in \mathbb{R}$ if and only if every subsequence of x_n has a sub-subsequence converging to x.

[1] Remember that $n! = n(n-1)(n-2)\cdots 2\cdot 1$