

HOMEWORK 3: DUE THURSDAY NOVEMBER 3 IN CLASS

Exercise I

Compute the following limits. Warning: some of them may not exist!¹ For this question, you only need write down the main steps without too much details. The last five are optional, but you are encouraged to solve them all.

1.

$$\lim_{x \rightarrow -\infty} \frac{2^x + 3^{x^2}}{x^2}$$

2.

$$\lim_{x \rightarrow 0} \frac{2\sqrt[3]{x} + \sin(x)}{\sqrt[3]{x} - \arctan(x)}$$

3.

$$\lim_{x \rightarrow +\infty} \frac{x^2 - 2\log(x)}{3x^2 + \log(x)}$$

4.

$$\lim_{n \rightarrow +\infty} \frac{4^{n+1} + 3^{n+1}}{5^n + 3^n}$$

5.

$$\lim_{x \rightarrow 1} \frac{x + \sin(x)}{\log(x)}$$

6.

$$\lim_{x \rightarrow 0} \frac{1 - 5^x}{1 - 3^x}$$

7.

$$\lim_{x \rightarrow 0} \frac{2x^2 + \tan(x^3)}{\sin^2(x)}$$

8.

$$\lim_{x \rightarrow 0} \frac{1 - \cos^2(x)}{\tan(x) \sin(2x)}$$

9.

$$\lim_{x \rightarrow 0} \left(1 - \frac{2}{7}x\right)^{\frac{1}{x}}$$

10.

$$\lim_{x \rightarrow 2} \frac{e^{x-2} - 1}{1 - \cos(x-2)}$$

11.

$$\lim_{x \rightarrow \infty} \frac{\log(x+2)}{\log(x+1)}$$

12.

$$\lim_{x \rightarrow 0} \frac{4\sin(x) - 2x\cos^2(x)}{3x\cos(x) - 5\sin(x)}$$

13.

$$\lim_{x \rightarrow 0} \frac{1 - \cos(x^4)}{\sin^2(x)}$$

14.

$$\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{\log\left(\frac{x}{x-1}\right)}$$

15.

$$\lim_{x \rightarrow 0} \log\left(2 - \frac{\sin^2(3x)}{\sin^3(\log(1+2x))}\right)$$

Exercise II

Prove that the limits of the following sequences do not exist by showing two subsequences with different limits.

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$$\lim_{n \rightarrow +\infty} \cos\left(n\frac{\pi}{6}\right)$$

•

$$\lim_{n \rightarrow +\infty} n^5 + (-1)^n n^8$$

Exercise III

- Let $f : [a, b] \rightarrow \mathbb{R}$ be a function satisfying the following: there exists a constant $C > 0$ and an exponent $\alpha \in (0, 1]$ such that for every $x, y \in [a, b]$ we have

$$|f(x) - f(y)| \leq C|x - y|^\alpha.$$

Prove that f is continuous.

- Find an example of a function f satisfying the above condition for $\alpha = 1$.
- For every value of $\alpha \in (0, 1)$ find a function f_α satisfying the above condition with exponent α .

¹i.e. left and right limits do not coincide