Random self-avoiding loops in Riemann surfaces — a direct construction of Werner's measure

Robert O. Bauer (University of Illinois at Urbana-Champaign)

In 1999, Oded Schramm introduced the Stochastic Loewner Evolution (SLE) to describe the scaling limit of loop-erased random walk and other random simple curves that arise as the scaling limit of interfaces in various 2 dimensional models of statistical mechanics at criticality, such as the boundaries of clusters in percolation or the Ising model. Since then, our understanding of such random simple curves has advanced significantly—rigorous convergence results linking various discrete models with continuous scaling limits, computation of critical exponents (e.g. Mandelbrot's conjecture), and a probabilistic explanation of (some of the) concepts, constructions, and results in Conformal Field Theory. For his contribution to these advances Wendelin Werner was awarded the Fields medal in 2006.

One of Werner's results is the construction of the unique conformally invariant restriction measure on self-avoiding loops in Riemann surfaces. In this talk I will give a direct construction of this measure based on chordal SLE(8/3). Our construction highlights some of the basic techniques for SLE-type curves—deriving uniqueness from very general properties, and obtaining invariant measures by integrating covariant measures. Throughout this talk, I will emphasize the topological and geometric aspects of the theory.