Brownian Motion on Lie Groups: Limits and Fluctuations in Large Dimensions

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Time **Thursday, Sept 25, 2014 at 14:00**
Place **Campus Kirchberg, room B04**

In this lecture, I will discuss the "strong law of large numbers" and the "central limit theorem" for these Brownian motions as $N \to \infty$. In the unitary case, one can make sense of these in terms of eigenvalues: the empirical spectral distribution forms a measure-valued process, which (as with the Hermitian case, given by the Dyson Brownian motion) converges almost surely to a measure-valued function, as shown by Biane in 1997. The fluctuations of this process are Gaussian, as proved by Lévy and Mäida in 2010. For non-normal matrices in $\mathbb{GL}_N$, the situation is much more difficult. By casting the question in a weak form, we can still understand the convergence of the Brownian motion as $N \to \infty$ as a "non-commutative measure" valued process. I will present some of my recent work describing this limit, solving a 20-year-old conjecture, and further discuss the fluctuations, which (as shown in recent joint work with Cébron) are also Gaussian.