The Malliavin Derivative Evaluated along a Stopped Path: applications, and generalization of the definition

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Place **Campus Kirchberg, room B04**

In this talk I will review some new results on martingale representation, when the driving process is either Brownian motion, fractional Brownian motion, or a pure jump process.

Suppose F is $\mathcal{F}_T$-measurable. In the Brownian case, the value of the martingale $E[F|\mathcal{F}_t]$ where $t \leq T$ can be calculated as the exponential of the time integral of one half times the second order Malliavin derivative of F evaluated along a stopped path. This is known in quantum mechanics as a *Dyson series*. In the other two cases, one obtains a slightly different representation, which also involves Malliavin derivatives. I will apply Dyson series to solve some previously unsolved problems in mathematical finance.

I will then try to address a question opened by this work. The Dyson series representation highlights a key difference between the Poisson case and the Brownian case. In the Poisson case, the Nualart-Vives-Malliavin derivative is a "global" derivative, in the sense that calculation of the Dyson series results in sampling the whole space. As a consequence, the Dyson series always converges. In the Brownian case, the Malliavin derivative (along a stopped path) is "local", and our initial construction of the Dyson series cannot be applied to random variables that are not infinitely Malliavin differentiable. For instance, the Black-Scholes formula can be represented as a Dyson series in the Poisson case, but not in the Brownian case. This begs the question: is there a useful generalization (in the sense of Hida distributions) of the definition of the Brownian Malliavin derivative under which the Dyson series converges in the non-smooth case? We will try to answer this question by using two different techniques: 1) Ito functional calculus, which guarantees the existence and uniqueness of viscous solutions of semilinear parabolic path-dependent partial dierential equations (PPDEs) under weak conditions 2) conditioning on the number of specific events in $[t, T]$ and the theory of Brownian excursions.