Stein Couplings for Concentration of Measure

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Abstract

For a nonnegative random variable Y with finite nonzero mean μ, we say that Y^s has the Y-size bias distribution if

$$E[Yf(Y)] = \mu E[f(Y^s)] \quad \text{for all smooth } f.$$

If Y can be coupled to Y^s having the Y-size bias distribution such that for some constant c we have $Y^s \leq Y + c$ almost surely, then Y satisfies the concentration of measure inequality

$$\max\left(\sup_{t\geq 0}\mathbb{P}(Y-\mu\geq t), \sup_{-\mu\leq t\leq 0}\mathbb{P}(Y-\mu\leq t)\right) \leq \left(\frac{\mu}{\mu+t}\right)^{(t+\mu)/c} e^{t/c}.$$

Applications of these bounds yield concentration results for the number of local maxima of a random function on a graph, urn occupancy statistics in multinomial allocation models, and the volume contained in k-way intersections of n balls placed uniformly over a volume n subset of d dimensional space. The two final examples are members of a class of occupancy models with log concave marginals for which size bias couplings may be constructed more generally.

Similarly, concentration bounds can be shown using the zero bias coupling, that is, when one can construct a bounded coupling of a mean zero random variable Y with finite nonzero variance σ^2 to a Y^* satisfying

$$E[Yf(Y)] = \sigma^2 E[f'(Y^*)] \quad \text{for all smooth } f.$$

Such couplings can be used to demonstrate concentration in Hoeffding's combinatorial central limit theorem under diverse assumptions on the permutation distribution.

The bounds produced by these couplings, which have their origin in Stein's method, offer improvements to those obtained by using other methods available in the literature.

This work is joint with Jay Bartroff, Subhankar Ghosh and Ümit Işlak.