Abstracts

On spectral characterizations of amenability

Claire Anantharaman-Delaroche

University of Orléans, France

Given a measured G-space (X, μ) and a probability measure m on G, we discuss the informations on the G-space X that are provided by the spectral analysis of the operator $\pi_X(m)$, where π_X is the unitary representation of G associated with (X, μ). Emphasis is put on characterizing the amenability of the G-action.

The heat kernel on symmetric spaces, fifteen years later

Jean–Philippe Anker

University of Orléans, France

At a previous conference (Luxembourg, September 1987), we conjectured an upper bound for the heat kernel on noncompact Riemannian symmetric spaces G/K. Our guess was based on explicit expressions available in some particular cases, namely when $\mathbf{rank}(G/K) = 1$, when G is complex or when $G = \mathrm{SU}(p, q)$.

In the meantime, this conjectural upper bound has been established in a rather large range and has proved to be a lower bound too. We shall give an overview of the present state of the subject, including recent joint work with P. Ostellari.

Divisible convex sets and prehomogeneous vector spaces

Yves Benoist

ENS Paris, France

A properly convex open cone in $\mathbf{R}^m$ is called divisible if there exists a discrete subgroup Γ of $\mathrm{GL}(\mathbf{R}^m)$ preserving C such that the quotient $\Gamma \backslash C$ is compact. We describe the Zariski closure G of such a group Γ.

It was known that this group G is reductive. We show that if C is divisible but is neither a product nor a symmetric cone, then Γ is Zariski dense in $\mathrm{GL}(\mathbf{R}^m)$. The main step is to prove that the representation of G in $\mathbf{R}^m$ is prehomogeneous.

Inducing and restricting unitary representations of nilpotent Lie groups

Hidenori Fujiwara

Kinki University, Japan

Let $G = \exp \mathfrak{g}$ be a connected, simply connected real nilpotent Lie group with Lie algebra $\mathfrak{g}$. Given an analytic subgroup $H = \exp \mathfrak{h}$ of G with Lie algebra $\mathfrak{h}$ and a unitary character χ of H, we construct the induced representation $\tau = \mathrm{ind}_H^G \chi$ of G. On the other hand, given an analytic subgroup $K = \exp \mathfrak{k}$ and an irreducible unitary representation π of G, we restrict π to K.

It is well known that there exists a strong parallelism between these two operations; inducing and restricting. We discuss this, focusing our attention on algebras of differential operators and a Frobenius reciprocity attached to these two procedures. Our study will be done in terms of the celebrated orbit method.

Extending positive definite functions from subgroups of locally compact groups

Eberhard Kaniuth

University of Paderborn, Germany

Let G be a locally compact group and H a closed subgroup of G. We call H an extending subgroup of G if every continuous positive definite function on H extends to some continuous positive definite function on G. Also, G is said to have the extension property when each closed subgroup of G is extending. The talk will first give a survey on what is known for some time regarding these properties and then focus on recent results for nilpotent groups.

Ramanujan complexes

Alex Lubotzky

University of Jerusalem, Israel

Ramanujan graphs are finite k-regular graphs the eigenvalues λ of their adjancency matrix satisfy $|\lambda| \leq 2\sqrt{k-1}$ or $\lambda = \pm k$. Examples were constructed as quotients of the tree associated to $PGL_2(K)$ when K is a non-archimedean local field.

The talk will describe a work in progress (jointly with B. Samuels and U. Vishne) on generalizing this concept from graphs to higher dimensional simplicial complexes, and constructions as quotients of the building associated with $PGL_d(K)$.

Bochner–Riesz means on the Heisenberg group and fractional integration on the dual

Detlef Müller

University of Kiel, Germany

Let L denote the sub–Laplacian on the Heisenberg group $\mathbb{H}_n$ and $T_r^\lambda := (1 - rL)_+^\lambda$ the corresponding Bochner–Riesz operator. Let Q denote the homogeneous dimension and D the Euclidean dimension of $\mathbb{H}_n$.

In joint work with D. Gorges, we prove a.e. convergence of the Bochner–Riesz means $T_r^\lambda f$ as $r \to 0$ for $\lambda > 0$ and for all $f \in L^p(\mathbb{H}_n)$, provided that

$$\frac{Q-1}{Q}\Big(\frac{1}{2} - \frac{\lambda}{D-1}\Big) \quad < \quad 1/p \quad \leq \quad 1/2.$$

Our proof is based on explicit formulas for the operators ∂_{ω^a}, $a \in \mathbb{C}$, defined on the dual of $\mathbb{H}_n$ by $\partial_{\omega^a}\hat{f} := \widehat{\omega^a f}$, which may be of independent interest. Here ω is given by $\omega(z,u) := |z|^2 - 4iu$ for all $(z,u) \in \mathbb{H}_n$.

Root graded Lie groups

Karl-Hermann Neeb

University of Darmstadt, Germany

Root graded Lie groups are (mostly infinite-dimensional) Lie groups whose Lie algebra is root graded in a topological sense. This means that it has a grading like a finite-dimensional simple complex Lie algebra by a finite irreducible reduced root system Δ, it contains the corresponding finite-dimensional simple complex Lie algebra, and it is generated by the root spaces. These Lie algebras are determined up to central extensions by the root system Δ and a coordinate structure. Root graded Lie algebras show up naturally in many geometric situations and also in mathematical physics, so that it is natural to ask for corresponding Lie groups. This problem is discussed in our lecture. We show that under natural assumptions on the coordinate structures there always is a centerfree Lie group corresponding to the centerfree Lie algebra for a given root system and coordinate structure. Then one has to face the problem to construct central extensions of these groups, which leads to interesting period maps relating K-groups and cyclic homology of topological algebras. In particular we obtain Lie group versions of Steinberg groups for algebras whose period maps satisfy a certain discreteness condition.

Hilbert bimodules associated to self-similar group actions

Volodia Nekrashevych

University of Kiev, Ukraine

We will talk about the Hilbert bimodules related to self-similar actions of groups and semigroups on the shifts of finite type. Basic examples of such groups and semigroups are the adding machines, the branch groups (like the Grigorchuk group), the groups related to aperiodic tilings, etc. The respective Cuntz-Pimsner algebras will be considered. Relations with hyperbolic dynamics and Thompson groups will be discussed.

Hierarchomorphisms of trees and combinatorial analogs of the group of diffeomorphisms of the circle

Yuri A. Neretin

University of Moscow, Russia

Let T be an inifinite tree, Abs be its boundary. A homeomorphism $q : Abs \to Abs$ is a hierarchomorphism if there exists a finite subtree $I \subset T$ such that q admits an extension to a map $T \setminus I \to T$. A group $Hier(T)$ of hierarchomorphisms contains the R. Thompson group and the group of locally analytical diffeomorphisms of the p-adic projective line. Properties of groups $Hier(T)$ are similar to properties of the group of diffeomorphisms of the circle. We discuss some constructions of unitary representations of $Hier(T)$.

Synthesis properties of orbits of compact groups

Detlev Poguntke

University of Bielefeld, Germany

The notion of sets of synthesis is best known in the case of $\mathcal{L}^1(G)$, G a locally compact abelian group. With each closed ideal I in $\mathcal{L}^1(G)$ there is associated a closed subset of the structure space $\mathcal{L}^1(G)\hat{} = \hat{G}$, namely the hull $h(I) = \{\chi \in \hat{G} \mid \ker_{\mathcal{L}^1(G)} \chi \supset I\}$. A closed subset A of $\hat{G}$ is called a set of synthesis if there exists exactly one closed ideal I in $\mathcal{L}^1(G)$ with $h(I) = A$; then I is necessarily equal to the kernel $k(A) := \bigcap_{\chi \in A} \ker_{\mathcal{L}^1(G)} \chi$ of A.

This notion (or these notions) can be generalized immediately to arbitrary Banach algebras, as soon as one agrees on the structure space to be considered. After recalling some known results on $\mathcal{L}^1$-algebras of nilpotent Lie groups, mainly where A is an orbit of a compact group acting homomorphically, we consider algebras of the following type:

Let K be a closed normal subgroup of a compact group L, and let Q be a symmetric semisimple involutive commutative Banach algebra, on which L acts. Suppose that L acts transitively on the Gelfand space $\hat{Q}$. Then one may form the generalized $\mathcal{L}^1$–algebra $B := \mathcal{L}^1(K, Q)$, which is endowed with a natural L–action.

As structure space $\hat{B}$ we take the collection of kernels of irreducible involutive representations of B (which coincides with the set $\mathrm{Priv}(B)$ of all primitive ideals) equipped with the Jacobson topology. It is shown that L–orbits in $\hat{B}$ are closed (which is not completely obvious!), and that they are sets of synthesis. Also the empty set is a set of synthesis, i.e., each proper closed ideal in B is contained in the kernel of an irreducible representation. The proof, briefly scetched, is an exercise in representation theory of compact groups.

Singular masas of von Neumann algebras: examples from the geometry of spaces of nonpositive curvature

Guyan Robertson

University of Newcastle, Australia

If Γ is a group, then the von Neumann algebra $VN(\Gamma)$ is the convolution algebra of functions $f \in \ell_2(\Gamma)$ which act by convolution on $\ell_2(\Gamma)$ as bounded operators. Maximal abelian $\star$-subalgebras (masas) of von Neumann algebras have been studied from the early days.

If Γ is a torsion free cocompact lattice in a semisimple Lie group G of rank r with no centre and no compact factors then the geometry of the symmetric space $X = G/K$ may be used to define and study masas of $VN(\Gamma)$. These masas are of the form $VN(\Gamma_0)$, where Γ_0 is the period group of some Γ-periodic maximal flat in X. There is a similar construction if Γ is a lattice in a p-adic Lie group G, acting on its Bruhat-Tits building.

Consider the compact locally symmetric space $M = \Gamma\backslash X$. Assume that T^r is a totally geodesic flat torus in M and let $\Gamma_0 \cong \mathbf{Z}^r$ be the image of the fundamental group $\pi(T^r)$ under the natural monomorphism from $\pi(T^r)$ into $\Gamma = \pi(M)$. Then $VN(\Gamma_0)$ is a masa of $VN(\Gamma)$. If in addition $diam(T^r)$ is less than the length of a shortest closed geodesic in M then $VN(\Gamma_0)$ is a *(strongly) singular masa* : its unitary normalizer is as small as possible. This result is part of joint work with A. M. Sinclair and R. R. Smith.

Free Group Representations and Their Realizations on the Boundary

Tim Steger

University of Sassari, Italy

Let Γ be a noncommutative free group on finitely many generators. We consider unitary representations of Γ which are weakly contained in the regular representations. Equivalently, these are the "tempered" representations, those whose matrix coefficients are almost in ℓ^2. Let Ω be the natural boundary of Γ. A representation which acts in a certain well-defined natural way on some L^2-space on Ω is called a *boundary representation.* All boundary representations are tempered. Conversely, if π is any tempered representation, there is an inclusion of π into some boundary representation. Such an inclusion is a *boundary realization* of π.

Consideration of examples leads to the *duplicity conjecture*: a given irreducible tempered representation has at most two inequivalent, irreducible boundary realizations. We give the details of this conjecture.

There are lots of representations of Γ, and one's intuition is that a "generic" representation is irreducibile. However, proving the irreducibility of a specific representation is usually difficult. In many cases, an analysis going by way of boundary realizations works. In certain cases one can prove simultaneously that a representation is irreducible and that it has exactly two inequivalent, irreducible boundary realizations; in others that it has exactly one boundary realization.

We sketch the construction of a large class of examples of irreducibile tempered representations of Γ. The construction is based on vector-valued multiplicative functions, and covers in a uniform way many of the previously studied examples.

Finally, we mention *Paschke's Conjecture*: if $f \in \ell^1(\Gamma)$ is of finite support, then there are at most finitely many irreducibile tempered representations π such that $\pi(f)$ has a nonzero kernel.

The Plancherel formula for real almost algebraic groups

Pierre Torasso

University of Poitiers, France

(joint work with M. S. Khalgui, Tunisia.)

In his lectures given at the University of Maryland during the special year held in 1982-83, M. Duflo stated a concrete Plancherel formula for real almost algebraic groups. We give a proof of it in the philosophy of the orbit method and following the lines of the one given in 1987 by M. Duflo and M. Vergne for simply connected semi-simple Lie groups.

Let G be our almost algebraic Lie group and $\mathfrak{g}$ its Lie algebra. The main ingredients of the proof are :

- the Harish-Chandra's descent method which, interpreting Plancherel formula as an equality of semi-invariant generalized functions, allows one to reduce it to such an equality on a neighbourhood of zero in $\mathfrak{g}(s)$, the centralizer in $\mathfrak{g}$ of any elliptic element s in G,

- the character formula near elliptic elements for the representations of the group constructed by M. Duflo, recently proved by the authors : roughly speaking, the character of a representation associated to a coadjoint orbit Ω is given, near an elliptic element s of G, by the Fourier transform of a canonical measure closely related to the Liouville measure on Ω^s, the set of s-fixed points in Ω,

- the Poisson-Plancherel formula near peculiar elliptic elements, the one said to be in good position. If s is such an element, this formula, generalizing the classical Poisson summation formula, states that the Fourier transform of an invariant distribution, which is the sum of a series of Harish-Chandra type orbital integrals of elliptic elements in $\mathfrak{g}(s)$, is a generalized function on $\mathfrak{g}(s)^*$ whose product by a Lebesgue measure is a tempered complex measure supported on the set of G-admissible and strongly regular forms contained in $\mathfrak{g}(s)^*$.

Property (T) and harmonic maps

Alain Valette

University of Neuchâtel, Switzerland

We plan to sketch the proofs of the following two results:

Theorem 1 (Y. Shalom, unpublished). Let G be a simple Lie group with finite centre, and maximal compact subgroup K. If G does not have property (T), then there exists a Hibert space-valued, non constant harmonic map $G/K \to \mathcal{H}$ which is equivariant with respect to an action of G on $\mathcal{H}$ by affine isometries.

Theorem 2. For $G = Sp(n,1)$ with $n \geq 2$, every harmonic G-equivariant map $G/K \to \mathcal{H}$ is constant.

The proof of Theorem 2 is based on recent ideas of M. Gromov. Altogether, these two results provide a new, geometric proof of property (T) for $Sp(n,1)$.

* * * * **

Harish-Chandra decomposition of Banach-Lie groups

Harald Biller

University of Darmstadt, Germany

The Harish-Chandra decomposition, a construction principle for unitary representations of semi-simple Lie groups on Hilbert spaces of holomorphic functions, is generalized to certain linear Lie groups of infinite dimension.

Haagerup property and spaces with walls

Pierre-Alain Chérix

University of Geneva, Switzerland

In the late seventies Haagerup proved that the length function associated with a free generating system of a non abelian free group is conditionnally negative definite. The existence of such a proper conditionnally negative definite function is one possible definition of Haagerup property. In the eighties, many different definitions of that property were introduced and were proved to be equivalent. Later Haaglund and Paulin introduced the notion of space with walls and of groups acting properly on such spaces. With Valette, they proved that a group acting properly on a space with walls has Haagerup property. The result of Haagerup can be translated directly in that context. It seems natural to ask whether the converse is true or not. In a joined work with Martin and Valette, we introduced a generalized notion of mesured space with walls and we proved that for countable groups,the converse is true. Namely, a countable group which has Haagerup property is acting properly on a measured space with walls.

Unitary duality, weak topologies and thin sets in locally compact groups

Jorge Galindo

University Jaume I de Castellón, Spain

Let G be a locally compact group with sufficiently many finite-dimensional representations (i.e. a MAP group). A general (probably exceedingly general) question is to what extent its finite-dimensional representations can be used to understand G.

In this talk we shall discuss some of the well-known cases of groups strongly determined by their finite-dimensional representations, such as Abelian or Moore groups, and some obstructions to a general theory, represented by van der Waerden or Kazhdan groups. The discussion will be based on unitary dualities, Bohr compactifications and thin sets as ways to relate a group to its finite-dimensional representations.

Turning to concrete results, we shall sketch joint work with Salvador Hernández characterizing the existence of Bohr compact subsets in a locally compact group G (that is, sets that are compact in the Bohr compactification bG of G) by means of the existence of I_0-sets in the sense of Hartman and Ryll-Nardzewski (a set $A \subseteq G$ is an I_0-set if every complex-valued function on A can be extended to an almost periodic function on G). This will be essential in proving that a locally compact group has no infinite I_0-sets if and only if it has at most countably many inequivalent irreducible finite-dimensional representations. A similar approach will be used to show that discrete goups always contain infinite weak Sidon sets in the sense of Picardello (a subset A of a locally compact group is a weak Sidon set if every complex-valued function on A can be extended to a function belonging to $B(G)$, the Fourier-Stieltjes algebra of G).

Fourier inversion on rank one compact symmetric spaces

Francisco Gonzalez

University of Lausanne, Switzerland

Conditions for the pointwise Fourier inversion of K-invariant functions using Cesàro means of a given order are established on rank one compact symmetric spaces G/K.

Spectral decomposition and discrete series representations on a p-adic group

Volker Heiermann

Humboldt University, Berlin, Germany

Topic of my talk will be the proof of a conjecture of A. Silberger on infinitesimal characters of discrete series representations of a p-adic group G. More precisely, I'll show the following: a cuspidal representation σ of a Levi subgroup L corresponds to the infinitesimal character of a discrete series representation of G, if and only if σ is a pole of Harish-Chandra's μ-function of order equal to the parabolic rank of L. The proof is by a spectral decomposition starting from a Fourier inversion formula analog to the Plancherel formula. This formula has been established previously in [*Une formule de Plancherel pour les éléments de l'algèbre de Hecke d'un groupe réductif p-adique,* Comm. Math. Helv. **76**, 388-415, 2001]. The results take part of my Habilitation thesis.

Integral Geometry and hypergroups

Grigori Litvinov

International Sophus Lie Center, Moscow, Russia
(The corresponding results are joint with M.I. Graev).

It is well known that the Radon Transform is closely related to the Fourier transform and harmonic analysis on the group of real numbers (or the additive groups of vectors in a finite dimensional real linear space). Similarly there are relations between some standard problems of Integral Geometry (in the sense of

Gelfand and Graev) and some commutative hypergroups (in the sense of J. Delsarte) and harmonic analysis on these hypergoups. This result can be treated as an answer for an old I.M. Gelfand's question on algebraic foundations of Integral Geometry.

Théorie des représentations et K-Théorie

Nicolas Prudhon

University Louis Pasteur, Strasbourg, France

Dans cet exposé, nous nous intéresserons aux liens entre la K-théorie des C^*-algèbres associées aux groupes de Lie semi-simples et ‡ la théorie des représentations de ces groupes.

Nous rappellerons tout d'abord quelques notions concernant la K-théorie des C^*-algèbres, puis nous expliquerons comment calculer la K-théorie de la C^*-algèbre réduite $C_r^*(G)$ d'un groupe de Lie semi-simple connexe G, via l'application d'indice de Connes-Kasparov, l'induc- tion de Dirac

$$\mu : R(K) \to K_*(C_r^*(G)) \quad ,$$

où K est un sous groupe compact maximal de G. Celle-ci est un ismorphisme, comme l'ont démontré A. Wassermann puis V. Lafforgue. Nous expliciterons sur l'exemple de $SL_2(\mathbf{R})$ le lien avec la théorie des séries discrètes. Ces résultats sont ‡ rapprocher avec ceux de Atiyah-Schmid sur la construction des séries discrètes sur le noyau L^2 d'un opérateur de Dirac.

Pour finir, nous nous intéresserons au calcul de la K-théorie de la C^*-algèbre maximale $C^*(G)$ d'un tel groupe G. Nous verrons que lorsque le groupe possède la propriété T de Kazhdan, la K-théorie de cette C^*-algèbre est différente de celle précédemment étudiée en ce sens que la représentation régulière

$$\lambda : C^*(G) \to C_r^*(G)$$

n'induit pas un ismorphisme en K-théorie (alors que c'est le cas par exemple pour $SL_2(\mathbf{R})$), et nous verrons comment la propriété T est "détectée" par l'induction de Dirac.

Property (T) and exponential growth of discrete subgroups

Jean-François Quint

ENS Paris, France

We recall the definition and properties of the indicator of growth of a discrete subgroup of a semisimple Lie group. In case the ambient Lie group has real ranq greater than 2, we apply results of H. Oh, related to property (T), to give controls on the indicator of growth.