Exercises in Algebraic Number Theory

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Sheet 10

To be handed in by: Friday, 8 January 2010, 2 pm.

1. (4 points) Let $(K, |\cdot|)$ be an ultrametric field. Let $(x_n)_{n\in\mathbb{N}}$ be a sequence of elements of K.
   (a) Show that $(x_n)_n$ is a Cauchy sequence if and only if $(x_{n+1} - x_n)_n$ is a sequence converging to 0.
   (b) Show that $(\sum_{i=1}^n x_i)_n$ is a Cauchy sequence if and only if $(x_n)_n$ converges to 0.
   (c) Show that $(\sum_{i=1}^n x_i)_n$ is a Cauchy sequence if and only if $(\sum_{i=1}^n x_{\sigma(i)})_n$ is a Cauchy sequence for all bijections $\sigma : \mathbb{N} \to \mathbb{N}$.

2. (4 points) Let K be a field and $|\cdot|_1, \dots, |\cdot|_n$ be pairwise inequivalent absolute values on K. Let $a_1, \dots, a_n \in K$. Show that for all $\epsilon > 0$ there is an $x \in K$ such that

$$|x - a_i|_i < \epsilon \quad \text{for all } i \in \{1, \dots, n\}.$$