Workshop on Graded Algebra and Geometry
December 5–7, 2012, University of Luxembourg

Titles and Abstracts

Alexander Alldridge. Minocourse: ‘*Supermanifolds of maps and fields*’

Abstract: We explain the definition of a new category of possible infinite-dimensional superspaces, which is well suited for the construction of supermanifolds of maps and fields. We will show the existence of such supermanifolds, and of superdiffeomorphism supergroups in the smooth and real analytic cases.

Tiffany Covolo. ‘*Higher Berezinian*’

Abstract: I will report on a recently published joint work with Norbert Poncin and Valentin Ovsienko. We defined the notions of trace, determinant and, more generally, Berezinian of matrices over a $(\mathbb{Z}_2)^n$-graded commutative associative algebra. The applications include a new approach to the classical theory of matrices with coefficients in a Clifford algebra, in particular of quaternionic matrices.

If time permits, I will also present some further developments: the cohomological interpretation of these graded notions, as well as first results obtained in the alternative categorical approach.

Rita Fioresi. ‘*Supersymmetric Spaces*’

Abstract: The geometric theory of supersymmetric spaces has a close relation with the Harish-Chandra modules, that is the representations of the pair consisting of a real supergroup G of classical type and its maximal compact (ordinary) subgroup. Such representations are realized in the Frechet superspace of the sections of certain line bundles over the quotient of the complexification of G modulo a borel subgroup. We are going to examine the general theory and to discuss the example of the super Siegel space. This work is a joint collaboration with C. Carmeli (University of Genova) and V. S. Varadarajan (UCLA).

Janusz Grabowski. ‘*Contact Courant algebroids*’

Abstract: I will present a systematic approach to contact and Jacobi structures on graded supermanifolds. In this framework contact structures are interpreted as symplectic principal GL(1)-bundles. Gradings compatible with the principal action lead to the concept of a graded contact manifold and that of a contact Lie algebroid, a contact analog of a Lie

algebroid. The corresponding cohomology operator is represented not by a vector field (de Rham derivative) but a first-order differential operator. Contact manifolds of degree 2 are studied as well as contact analogs of Courant algebroids.

Dimitry Leites. Minicourse: '*Unexpected supersymmetries*'

ABSTRACT: Being described even briefly, in the abstract, these supersymmetries will not be unexpected anymore, but I surrender to the requirements of the organizers to reveal at least a summary. Here are the main points I hope to cover:

1) Sophus Lie introduced the groups and algebras that now bear his name to describe symmetries of differential equations. Elie Cartan suggested two types of definitions of differential equations: as subspaces in the spaces of jets, and in terms of exterior differential forms. The second approach immediately shows that every differential equation possesses a supergroup of symmetry (perhaps, trivial, reducing to a group). This observation makes the findings of Wess and Zumino (and earlier pioneers of SUSY) not as astounding as it is still considered. To investigate classical equations of mathematical physics from this point of view is an interesting open problem.

2) Towards "non-commutative geometry", i.e., spaces ringed by non-commutative rings or algebras. I intend to say why I interpret the results of the organizers as one of the most remarkable achievements towards the construction of non-commutative geometry, and why Neklyudova's theorem does not ban studies of graded-commutative algebras, as I thought before.

3) Speaking of manifolds and supermanifolds mathematicians consider real and complex ones (or, lately, over p-adic numbers). The Minkowski superspaces introduced by physicists are, however, neither real nor complex. They are a certain mixture of both in presence — this is vital — of a non-holonomic, i.e., non-integrable, distribution. I intend to describe invariants of such "real-complex" supermanifolds and give results of computations for the Minkowski superspaces and supercurves.

4) Lie superalgebras appeard not in 1970s in works of physicists, as some think. They appeared in 1930s, in topology, mainly over finite fields. Classification of simple Lie algebras over algebraically closed fields of characteristic p>7 was conjectured in 1969 by Kostrikin and Shafarevich and recently proved, with amendments, mainly by efforts of Premet and Strade who relied on results of Block and Wilson, and several other researchers. In characteristics 3 and, especially, 2 only disjoint examples not fitting into the KSh procedure were known until recently. I intend to formulate a conjecture for these cases embracing all known examples. It already helped to find several new simple Lie algebras. In its formulation main ingredients include non-holonomic distributions and — for p=2 — Lie superalgebras as hidden supersymmetries of Lie algebras.

Dmitry Roytenberg. '*Superalgebras of smooth functions and derived geometry*'

Abstract: Homotopy-commutative rings play the same role in derived algebraic geometry as commutative rings do in ordinary algebraic geometry. Modeling homotopy-commutativity in general is complicated; however, over a ground field k of characteristic zero, differential graded k-algebras provide an adequate model. Every other type of geometry (analytic, C-infinity, ...) has its own underlying commutative algebra, involving more structure than just a commutative ring. The derived version of such a geometry requires an "up to homotopy" version of the corresponding algebra. We show that, when the ground field is of characteristic zero, the "differential graded" (in a suitable sense) version of the commutative algebra suffices to model the corresponding derived geometry. This is joint work with David Carchedi.

Vladimir Rubtsov. '*Double brackets on associative algebras and Poisson structures on their representations*'

Abstract: We discuss double structures in sense of M. Van den Bergh on free associative algebras focusing on the case of quadratic Poisson brackets. We establish their relations with an associative version of Yang-Baxter equations, we study a bi-hamiltonian property of the linear-quadratic pencil of the double Poisson structure and propose a classification of the quadratic double Poisson brackets in the case of the algebra with two free generators. Many new examples of quadratic double Poisson brackets are proposed.

Vera Serganova. '*Associated varieties for algebraic supergroups and fiber functor*'

Abstract: The category of representations of a semi-simple algebraic supergroup over a field of characteristic zero is not semi-simple. In many ways the situation is similar to the classical case in positive characteristic.

Some time ago M. Duflo and I constructed a functor from this category to the category of equivariant quasicoherent sheaves on the cone of self-commuting odd elements of the corresponding Lie superalgebra.

The geometric fiber of the corresponding sheaf at a generic point of the associated variety defines a functor from a block of the original category to a semi-simple block in the category of representations of a supergroup of the same type of smaller rank. I discuss applications of this fiber functor in representation theory: proof of Kac-Wakimoto conjecture and connection with modified dimension defined by N. Geer, J. Kujawa and B. Patureau-Mirand. I also formulate open questions and conjectures about the fiber functor.

Pavol Ševera. '*Integration of differential graded manifolds*'

Differential graded manifolds can be seen as higher Lie algebroids. I will describe how to integrate them to higher Lie groupoids: 1. For any dg manifold we can construct a simplicial set, which turns out to be a Kan simplicial Banach manifold. 2. By imposing a gauge condition, we can locally find a finite-dimensional Kan submanifold, which can be seen as a

local Lie n-groupoid. These local n-groupoids are (non-uniquely) isomorphic on the overlaps. I will mention many open problems. Based on a joint work with Michal Siran.

Marco Zambon. '*Homotopy moment maps*'

ABSTRACT: The notion of moment map is central in symplectic geometry, where the functions on the symplectic manifold (the "observables") form a Lie algebra. We extend this notion to higher differential forms, defining a homotopy moment map to be an L_∞-algebra morphism into the observables. We give a cohomological interpretation (which provides a natural notion of equivalence), show that certain equivariant cocycles induce homotopy moment maps, and discuss uniqueness issues. This is joint work with Chris L. Rogers (Göttingen) and Yael Fregier (MIT).