Abstracts for the Workshop: Computations with Modular Forms

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Andrew Booker: Computing non-holomorphic modular forms I will describe some of the theory and practice of computing non-holomorphic modular forms, for both GL(2) and GL(3). This will cover recent joint work with Andreas Strombergsson and my student, Ce Bian.

Gebhard Böckle: Drinfeld cusp forms and their combinatorics We begin with a brief general introduction to Drinfeld modular forms indicating some analogies with classical modular forms. Our main aim will be to explain Teitelbaum's combinatorial description of such forms as harmonic cochains which are invariant under certain finite index subgroups of $\mathrm{GL}_2(\mathbb{F}_q[t])$. From this description one obtains an algorithm which allows the computation of Hecke eigensystems attached to Drinfeld modular forms. Some basic open questions will be explained.

Johan Bosman: Computing Galois representations mod p In this talk we will discuss the work by Bas Edixhoven, Jean-Marc Couveignes, Robin de Jong, Franz Merkl and the speaker on computing coefficients of modular forms, such as the tau function that gives the coefficients of the modular discriminant. The computation goes via computation of mod ℓ representations, that can be encoded in terms of polynomials. Theoretically this leads to an algoritm that on input a prime number p can compute $\tau(p)$ in time polynomial in $\log(p)$. In the talk we will mainly discuss what we can compute in practice and show some nice applications of the computations.

Ralf Butenuth: Computing Hecke Operators On Drinfeld Cusp Forms We use Teitelbaum's description of Drinfeld cusp forms as harmonic cochains to compute Hecke operators on these forms. We will explain the technical details of the algorithm and we will present some computational examples.

Craig Citro: Enumerating mod p Hecke eigensystems (Joint with Alex Ghitza.) Given a level N and a prime p, one knows by work of Serre and Tate that there are only finitely many eigensystems (i.e. systems of Hecke eigenvalues) coming from mod p modular forms of level N. By the recent proof of Serre's Conjecture, this is equivalent to determining all odd two-dimensional mod p Galois representations. I'll discuss an algorithm for doing this at level 1, as well as its implementation (in Sage).

John Cremona: Computing modular forms over imaginary quadratic fields I will give a survey of nearly 30 years' work on the explicit computation of cusp forms of weight 2 and arbitrary level over imaginary quadratic fields, with a view towards making explicit and gathering evidence to support a Shimura-Taniyama-Weil-type correspondence with isogeny classes of elliptic curves. The work has formed the basis of four PhD theses (including my own) so far, and includes implementations in APL, Algol68, C++ and Magma, some of which no longer runs. In an accompanying Software Presentation I will say more about the available implementations and demonstrate those which do run, and discuss plans for updating an unifying all the past work in Sage.

Steve Donnelly: Modular Forms in Magma This survey of the principal algorithms used in Magma routines for computing modular forms will focus on areas that are currently under development, including half-integral weight forms and Hilbert modular forms.

Frazer Jarvis: Fermat's Last Theorem Over $\mathbb{Q}(\sqrt{17})$? We discuss generalisations of the work of Ribet and Wiles to real quadratic fields. We explain that much of the numerology required for a fairly direct generalisation of the proof of Fermat's Last Theorem works for the two fields $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt{17})$, reducing the problem to the non-modularity of Frey curves. While this works well for $\mathbb{Q}(\sqrt{2})$, Dembélé has produced a cusp form over $\mathbb{Q}(\sqrt{17})$ whose existence prevents us from deducing the final contradiction for $\mathbb{Q}(\sqrt{17})$; more thought will be needed in this case.

Ian Kiming: Modular mod p^n representations: Theory and computation We have initiated a study of modular Galois representations 'mod p^n', particularly the question of stripping powers of p from the level. In the talk I will first discuss a little the basics of the theory of such representations, and go into some of the details of a first theorem on level reduction, namely that the level can be made prime to p. In the proof we utilize, among other things, deep theories due to Katz and Hida. After this, I will report on some new theorems that relate to the problem of detecting congruences mod p^n between newforms (of different weights), and also report on some numerical experiments. This last part is still in progress.

This is joint work with Imin Chen (SFU, Burnaby, Canada) and Jonas B. Rasmussen (Copenhagen).

Morten Larsen: Systems of Hecke eigenvalues in cohomology We will for a general number field consider how to get an action of the Hecke algebra on cohomology groups, and show that if a system of Hecke eigenvalues in cohomology groups with admissible coefficient module, then the system will also occur in a cohomology group with 1-dimensional coefficient module. This relies on methods by Ash and Stevens concerning double coset algebras.

David Loeffler: Approaches to computing overconvergent p-adic modular forms I will give a brief introduction to the definition of the space of overconvergent modular forms, which is in a sense a p-adic 'completion' of the space of classical modular forms, and describe two approaches to computing these objects: via explicit parametrisations of modular curves, and via the Eichler-Selberg trace formula. This is a survey of work by several authors, including Kevin Buzzard, Frank Calegari, Lloyd Kilford and myself. I will also present implementations of both of these in PARI/GP.

Ken McMurdy: Eta Products and Models for Modular Curves Eta products generate a family of modular forms which can be incredibly useful when obtaining explicit equations for modular curves and the various maps between them. The primary advantage of using eta products is that they are supported on the cusps, and their q-expansions at any cusp can be easily computed. After reviewing Ligozat's criteria for determining when an eta product gives a legitimate modular form, I will show how all of the q- expansions of an eta product may be computed using families of Tate curves. I will also discuss some ways to get around the problem of not having enough eta products, as in the case of $X_0(p)$.

Nathan Ryan: Computing with Siegel Modular Forms of Degree 2 Siegel modular forms of degree 2 and level 1 are most explicitly computed via work of Igusa and Skoruppa. In this talk we survey their results and discuss some recent computations. In joint work with Lauren Grainer, I've verified the Riemann hyopthesis for the standard and spinor L-functions. In joint work with Kevin McGoldrick, I've formulated an analogue of the Sato-Tate conjecture for Siegel modular forms. Finally, in joint work with Ralf Schmidt, Sharon Garthwaite and David Farmer, I've verified the functional equation of a degree 10 L-function as predicted by Langlands. If time permits, I will discuss what is known about computing Siegel modular forms of nontrivial level.

Nathan Ryan: Siegel Modular Forms in Sage [implementation presentation] We give a demonstration of recent Sage code written by Nils Skoruppa, Craig Citro and me. We also briefly discuss the implementation, making special mention of the compiled code written in Cython.

Mehmet Sengün: Computation of modular forms for imaginary quadratic fields using group cohomology We will present an algorithm which uses group cohomology to compute modular forms over imaginary quadratic fields. Time permitting we will discuss problems and conjectures concerning the arithmetic of these forms.

Nils Skoruppa: On the computation of modular forms of half integral weight We present a method to generate, for a given level, character and half integral weight, closed formulas for the Fourier coefficients of a basis of the corresponding space of modular forms. We present the idea leading to this method, and we discuss possible refinements, consequences and how this can be turned into a ready to compute algorithm. In this talk we concentrate mainly on the case of weight 3/2.

Fredrik Strömberg: Computational aspects of non-holomorphic automorphic forms on $PSL(2,\mathbb{R})$ I will talk about computations of non-holomorphic automorphic forms on subgroups of $PSL(2,\mathbb{R})$. In particular I will discuss the „automorphy method“ (or „Hejhal's method“) which is a general method to compute automorphic forms numerically.

The original implementation by Hejhal was used to compute Maass forms on $PSL(2,\mathbb{Z})$. The method's simplicity allows for many generalizations, e.g. more general Fuchsian groups and representations as well as more general types of automorphic forms.

I will present some of the situations where this method has been successfully implemented: Maass forms on subgroups of $PSL(2,\mathbb{Z})$ with characters and multiplier systems, non-holomorphic Eisenstein series, automorphic Green's functions and harmonic weak Maass forms.

Maria Teider: Computing number theoretical operators using Magma tools for lattices Using the Kneser's neighbour method for lattices, we efficiently compute Hecke operators on theta series (an important class of examples of Siegel modular forms). The algorithm implementation in Magma is demonstrated.

Rebecca Torrey: Modular forms and mod ℓ Galois representations over imaginary quadratic fields In a forthcoming paper, Buzzard, Diamond and Jarvis give a generalization of the refined version of Serre's conjecture to totally real fields. In particular they give an explicit recipe for possible weights of modular mod ℓ Galois representations. I will discuss their conjecture and my subsequent investigation of the relationship between modular forms and mod ℓ Galois representations over imaginary quadratic fields.

Helena Verrill: Fundamental domains and noncongruence subgroups I will describe an algorithm to determine whether a subgroup of $\mathrm{SL}_2(\mathbb{Z})$ (given by a fundamental domain and set of generators given by specifying matrices identifying edges) is a congruence subgroup. This is very similar to the algorithm of Lang, Lim, Tan based on the results of Kulkarni and Wohlfarth, however, in their case the subgroups were of $\mathrm{PSL}_2(\mathbb{Z})$, so some minor modification is required.

John Voight: Computing fundamental domains for cofinite Fuchsian groups We present an algorithm to compute a Dirichlet domain for a cofinite Fuchsian group Γ which also produces a finite presentation for the group Γ. We discuss applications of this algorithm to arithmetic, computing group invariants, group cohomology, and modular and automorphic forms. Along the way, we will give many pictures and examples to illustrate how the algorithm works.

Lynne Walling: Siegel modular forms, Hecke operators, and L-series Siegel introduced generalised theta series to study how often a given quadratic form represents other quadratic forms. These gave us our first examples of Siegel modular forms, which are analytic functions in a symmetric, $n \times n$ variable, transforming under (a subgroup of) the symplectic group. For each prime p, there are $n+1$ basic Hecke operators (n of which are algebraically independent). We will describe their action of Fourier coefficients and on Siegel theta series. We will also look at L-series associated to Siegel modular forms, particularly when $n = 2$, and look at how well these capture the Fourier coefficients of the modular form.

Jared Weinstein: Computing the local components of a cusp form Cuspidal eigenforms f are one and the same as cuspidal automorphic representations π of the adele group $\mathrm{GL}_2(\mathbf{A}_\mathbf{Q})$. Given such an f, can one compute the local components π_p of the associated π? This is the same as determining the local Galois representation of f at p. The question is trivial for $p \nmid N$: in that case the Fourier coefficient a_p determines π_p. But when $p \mid N$ and especially for $p^2 \mid N$ there can be very interesting behavior. We demonstrate how π_p can be computed using modular symbols.

Dan Yasaki: Hecke operators and Hilbert modular forms Let F be a real quadratic field with ring of integers O and with class number 1. Let Γ be a congruence subgroup of $\mathrm{GL}_2(O)$. We describe a technique to compute the action of the Hecke operators on the cohomology $H^3(\Gamma; \mathbb{C})$. For F real quadratic this cohomology group contains the cuspidal cohomology corresponding to cuspidal Hilbert modular forms of parallel weight 2. Hence this technique gives a way to compute the Hecke action on these Hilbert modular forms.